

48th CIRP Conference on MANUFACTURING SYSTEMS - CIRP CMS 2015

## Planning of modular fixtures in a robotic assembly system

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### Abstract

Increasing product varieties is beneficial for companies in terms of expanding the market and harmful in terms of increasing manufacturing costs. Designing and fabricating different fixtures for producing different products with different geometries is a significant portion of the manufacturing costs. To overcome to this problem in a mid-volume mid-variety robotic assembly system, an optimization model is developed to minimize hole-pattern modular fixtures' preparation time and efforts. Using this model, the best locations for placing different products and jiggling-pins are determined, considering all possible part's translations and rotations on the holder. The model is solved by GAMS using the BARON solver for different examples to prove the efficacy of the proposed model.

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Peer-review under responsibility of the scientific committee of 48th CIRP Conference on MANUFACTURING SYSTEMS - CIRP CMS 2015

**Keywords:** Modular fixtures; Mathematical programming; Optimization; Rotation; Translation; Nonlinear model

### 1. Introduction

In mid-volume mid-variety manufacturing systems, a significant portion of manufacturing costs are related to designing and fabricating different fixtures. Looking to the literature, the fixtures' associated costs are about 10 to 20 percent of the total manufacturing costs [1], [2]. Increasing the variety of products which are produced in a manufacturing system increase the fixtures cost. In today's competitive world mass production of a few products is no longer efficient. Because manufacturers are faced with different customers in a wide geographic region who presumably have different needs and expectations, they need to provide them with different products and features. Otherwise the company will lose its market share in different regional and international markets. Also, firms are faced with continuous changes in customers' needs and they have to make their customers satisfied through offering variety of products and features. With emerging new materials and technologies, products' life cycles are reduced, intensifying the competition among manufacturers to produce greater varieties of products and services [1].

Therefore, increasing the variety of products is a solution for manufacturers to penetrate different markets and win more

market share. But they should be aware that managing product varieties is very challenging and more attention must be given to the whole production cycle. One of the biggest obstacles for these companies is their frequent need to design different fixtures, which increases manufacturing costs dramatically and may decrease firms' competitiveness.

To overcome this problem and to reduce fixtures' associated costs, many researchers work on designing flexible fixtures which can hold different products' geometries and, as a result, increase sustainability of manufacturing systems by reducing the required number of fixtures [3–8]. Kang and Peng [5] believe that designing flexible fixtures can reduce 80% of fixture associated costs.

There are different approaches in designing flexible fixtures, such as sensory-based techniques, modular and reconfigurable fixtures, programmable conformable clamps, phase change fixtures, and adaptable fixtures. In figure 1 and 2, a modular fixture and a programmable clamps fixture are depicted.

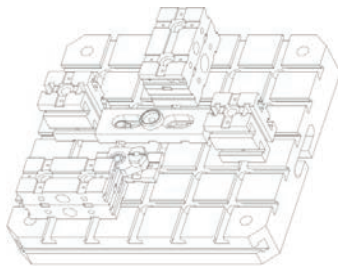


Fig. 1. A modular fixture structure [9]

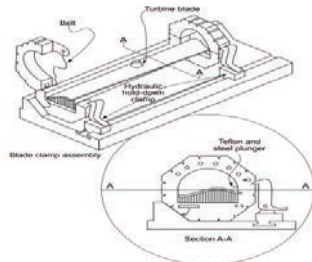


Fig. 2. A programmable clamps fixture [6]

Among the different types of flexible fixtures, modular fixtures are one of the most important ones which are widely used in different industries [5], [8], [10]. Modular fixtures provide firms with more flexibility and also enable them to fix irregular shapes [11]. They also reduce productions' lead time by easily adapting to different workpieces with different shapes and sizes [5].

In this paper, modular fixtures are used in a make to order production system in order to hold a variety of geometrically different products in a robotic assembly system. These cradles are installed on top of a set of cradle in a "Hold'n Go" conveyor-belt loop moving through the assembly system. The modular cradle has a hole-pattern on its adapter plate for jiggling-pins to be inserted. The designed fixture is shown in figure 3.

This solution brings universality, because pins can be easily re-arranged in order to fix a wide variety of part geometries by providing sufficient support reactions for parts and prevent slippage and shaking during assembly processes. For instance, it is shown in figure 4 how an irregular shape is fixed by inserting four pins. These four pins secure the part in place for extreme impact force conditions from any direction.

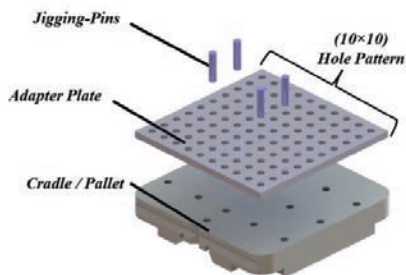


Fig. 3. The designed hole-pattern modular fixture

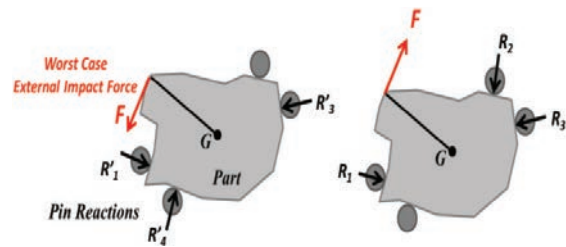


Fig. 4. Fixing an irregular shaped workpiece by inserting 4 pins

These jiggling pins can be re-arranged in order to fix a wide variety of part geometries. However, the larger the number of changed pins, the longer the preparation time for fixtures. Therefore, in this study a mathematical model is developed to find the best pins' location in order to minimize the fixtures' preparation time. For this purpose, the number of pins' replacements must be minimized. The developed optimization model enables the system to determine the best locations for placing different workpieces on the cradles and the best locations for inserting pins to fix them. Note that manually enumeration of different possible pins' locations for different workpieces is very hard, and by increasing the problem's size, it becomes impossible. The formula for computing the number of different combinations is presented in Section 3.

The remainder of this paper is organized as follows: Section 2 contains a precise statement of the problem and formulation. In section 3, numerical examples are provided and solved in order to prove the efficacy of the proposed model, and finally, conclusions are drawn and future works proposed in last sections.

## 2. Methodology

To minimize the time and efforts associated with designing and applying different fixtures in a make to order the automated assembly system, a modular fixture is designed. A robot is placed on top of the conveyor loop to place different workpiece on the cradle and fixing them by inserting jiggling pins. Replacements of pins are controlled by a mathematical model. In this section, research problem is described and then the mathematical model will be proposed.

### 2.1. Problem Statement

As mentioned earlier, the research question is finding the best possible places for pins to be inserted in a universal modular cradle for a set of different geometries parts in order to minimize the number of pin replacements. Note that it is assumed in this paper that the possible pins' locations for each part are predetermined by checking the force closure equations. Therefore, finding the best locations among these possible locations is considered. The proposed model takes all the workpiece translation and rotation into consideration to find out the best parts and pins location (figure 5).

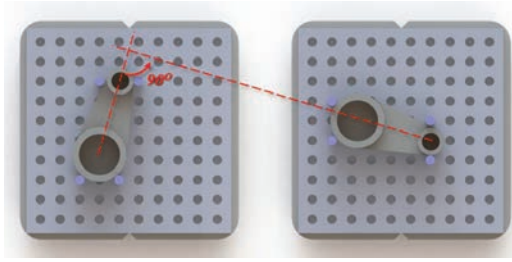


Fig. 5. Changing pins' location by 90° rotation

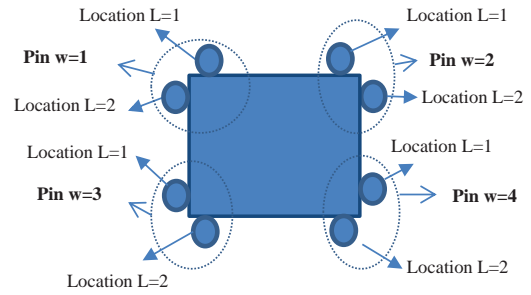


Fig. 6. Introducing different indexes

2.2. Mathematical Model

The mixed integer non-linear programming (MINLP) model is developed using the following notations:

<b>Index:</b>	
$i$	part's number ( $i = 1, 2, \dots, I$ )
$w, w'$	pin's number ( $w = 1, 2, \dots, W$ )
$l$	$l$ 'th location for a pin ( $l = 1, 2, \dots, L$ )
<b>Parameters:</b>	
$px_{iwl}$	the first coordinate (X) of pins' possible locations for part $i$ , pin $w$ and $l$ 'th place
$py_{iwl}$	the second coordinate (Y) of pins' possible locations for part $i$ , pin $w$ and $l$ 'th place
$M'$	a big number(1000)
<b>Variables:</b>	
$M_{iwl}$	Binary variable. $M_{iwl} = 1$ only if the corresponding coordinates are selected. Elsewhere $M_{iwl} = 0$ .
$n_{iww'}$	Binary variable. $n_{iww'} = 1$ only if the corresponding pin does not change for two different successive parts. elsewhere $n_{iww'} = 0$
$x_{iwl}$	the first coordinate (X) of the chosen location
$y_{iwl}$	the second coordinate (Y) of the chosen location
$x'_{iw}$	the first coordinate (X) of the chosen location for $i$ 'th part and $w$ 'th pin
$y'_{iw}$	the second coordinate (Y) of the chosen location for $i$ 'th part and $w$ 'th pin
$zx_{iww'}$	horizontal distance between two pin locations for two successive parts, part $i$ and $(i + 1)$ .
$zy_{iww'}$	vertical distance between two pin locations for two successive parts, part $i$ and $(i + 1)$ .
$nr_i$	The translation of part $i$ to right
$nl_i$	The translation of part $i$ to left
$nu_i$	The translation of part $i$ to up
$nd_i$	The translation of part $i$ to down
$\theta_i$	The rotation coefficient of part $i$ around the origin point (0,0)

To clarify different indexes, they are shown in the figure 6 for a simple problem. In this example  $w$  and  $l$  are assumed to be 4 and 2 respectively. This means that four different places must be chosen for pins to be inserted and each pin has two different possible locations.

As mentioned before, in this study it is assumed that the possible sets for pins' locations are predetermined in a way that provides sufficient contact with parts and prevent parts' slippage during assembling process. The possible set for each part is given in two separate matrices:  $px_{iwl}$  and  $py_{iwl}$  for X and Y coordinates.

In this section the developed mathematical model is presented:

$$obj = \max(\sum_i \sum_w \sum_{w'} n_{iww'}) \tag{1}$$

Subject to:

$$\sum_l M_{iwl} = 1 \quad \forall i, w \tag{2}$$

$$x_{iwl} = px_{iwl} * M_{iwl} \quad \forall i, w, l \tag{3}$$

$$y_{iwl} = py_{iwl} * M_{iwl} \quad \forall i, w, l \tag{4}$$

$$x'_{iw} = \sum_l x_{iwl} * \cos(\theta_i * \frac{\pi}{2}) - \sum_l y_{iwl} * \sin(\theta_i * \frac{\pi}{2}) + nr_i - nl_i \quad \forall i, w \tag{5}$$

$$y'_{iw} = \sum_l x_{iwl} * \sin(\theta_i * \frac{\pi}{2}) + \sum_l y_{iwl} * \cos(\theta_i * \frac{\pi}{2}) + nu_i - nd_i \quad \forall i, w \tag{6}$$

$$0 \leq x'_{iw} \leq 10 \quad \forall i, w \tag{7}$$

$$0 \leq y'_{iw} \leq 10 \quad \forall i, w \tag{8}$$

$$0 \leq \theta_i < 4 \quad \forall i \tag{9}$$

$$zx_{iww'} \geq x'_{iw} - x'_{(i+1)w'} \quad \forall i, w, w' \tag{10}$$

$$zx_{iww'} \geq x'_{(i+1)w'} - x'_{iw} \quad \forall i, w, w' \tag{11}$$

$$zy_{iww'} \geq y'_{iw} - y'_{(i+1)w'} \quad \forall i, w, w' \tag{12}$$

$$zy_{iww'} \geq y'_{(i+1)w'} - y'_{iw} \quad \forall i, w, w' \tag{13}$$

$$zx_{iww'} + zy_{iww'} \leq M' * (1 - n_{iww'}) \quad \forall i, w, w' \tag{14}$$

$$M_{iwl}, n_{iww'} \in \{0,1\} \tag{15}$$

$$zx_{iww'}, zy_{iww'} \geq 0 \tag{16}$$

$$x_{iwl}, y_{iwl}, x'_{iw}, x'_{iw}, nl_i, nr_i, nd_i, nu_i, \theta_i \in int \tag{17}$$

The first equation is the objective function, which is maximizing the number of fixed pins between two successive parts. Note that in this paper instead of minimizing the number of pin changes after each pins' rearrangement, the number of fixed pins are maximized. Equation 2 states that the model must choose one of the possible locations for each pin.  $M_{iwl} = 1$  if the corresponding location is chosen and  $M_{iwl} = 0$  elsewhere. Equations 3 to 6 calculate the pins' location on XY axis. It should be considered that equations 5 and 6 are the key equations which fulfil the parts' translation and rotation. In this model, four different rotation angles ( $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ ) are considered, because the other angles cause different challenges in the assembly line and also cause unfeasible solutions. To clarify the rotation and translation equation, look at the following example.

A rectangle with 4 coordinates, (2,1), (2,3), (6,1) and (6,3) is assumed. In this case, the rectangle rotates  $\alpha = \frac{\pi}{2}$  around the XY axis origin and translates 3 points to up ( $nu = 3$ ) and 8 points to right ( $nr = 8$ ). The results are shown in figure 7. It should be noted that after the rotation and without any translation, the result is an infeasible solution because the rectangle is out of 10X10 feasible regions.

Equations 7 and 8 state that the final x and y coordinates must be in the feasible region. Equation 9 states that the rotation coefficient must be chosen from this set: {0,1,2,3}. Equations 7 to 11 calculate number of fixed pins between two successive parts. Equations 7 and 8 are separated from the following constraint:

$$zx_{iww'} > |x'_{iw} - x'_{(i+1)w'}| \tag{18}$$

Because  $zx_{iww'}$  is a positive variable, if two points  $x'_{iw}$  and  $x'_{(i+1)w'}$  don't match with each other, then  $zx_{iww'} > 0$ . The same is true for equation 9 and 10; if  $y'_{iw}$  and  $y'_{(i+1)w'}$  don't match with each other, then  $zy_{iww'} > 0$ . If two points completely match each other (same X and Y) then  $zx_{iww'}$  and  $zy_{iww'}$  could get the value 0 or more than 0. On the other hand, equation 11 forces  $n_{iww'}$  to get value 0 if the distance between two points is more than 0. In other words, if x and y coordinates for two points don't match each other, variable  $n_{iww'}$  is forced to get a 0 value. Otherwise it may get value 0 or more than 0. Because  $n_{iww'}$  is in the objective function with the maximization sign,  $n_{iww'}$  gets 1 when there is no force to get value 0.

In this model equation 5 and 6 are non-linear, so the whole model will be non-linear. Therefore, the model is solved by GAMS using the BARON solver.

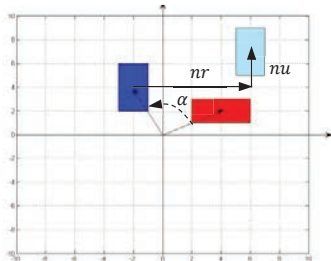


Fig. 7. Rotation and translation of workpiece on the cradle

### 3. Numerical Examples

To compare the results and show the efficacy of the proposed model, another model will be presented by replacing equation 5 and 6 with new equations. In this new model, by replacing these two equations by following equations, both translation and rotation flexibility will be eliminated and the result will be a basic model which can only find the best pins' location among the predetermined possible locations. The new equations are:

$$x'_{iw} = \sum_l x_{iwl} \quad \forall i, w \tag{19}$$

$$y'_{iw} = \sum_l y_{iwl} \quad \forall i, w \tag{20}$$

The new model which is formed by replacing these two equations by equation 5 and 6 will be called the basic model.

In this section, to show the applicability and efficacy of the proposed models, three different problems with different sizes are solved. The problem solution space is very large even for the basic model. In the basic model, since one of the possible locations for each pin must be chosen, the number of possible combinations will be equal to  $\binom{l}{1}^w$ . Having i different workpieces the number of possible combination will be  $\binom{l}{1}^{w \cdot i}$ . Therefore, for a small size problem with 5 different workpieces, four jiggging-pins ( $w=4$ ), and two different possible location for each pin ( $l=2$ ), the number of feasible solutions will be  $\binom{l}{1}^{wi} = 2^{20} = 1,048,576$ . It should be mentioned that feasible solution space is grown exponentially by increasing the parameters' value. For instance, the number of possible combinations for 10 different parts and the same value for w and l will be equal to  $\binom{l}{1}^{wi} = 2^{40} \sim 1,099,511,627,776$ .

In this section, 3 different problems with different sizes are presented. In these problems 5, 10 and 50 different workpieces must be planned for fixing on the cradle respectively. To investigate the efficacy of the proposed model the solutions are compared by basic models. It should be pointed out that the both models are coded in GAMS but the first model is solved by the BARON Solver based on its non-linearity nature, and the basic model is solved by Cplex Solver. As mentioned earlier, in the first problem, the robot should be planned to fix 5 different workpieces on the fixture one after another in order to minimize the number of pin's replacements. The problems' indices are given below:

$$i = \{1,2, \dots, 5\}, w = \{1,2,3,4\}, l = \{1,2\}$$

The problem's parameters are given in table 1 and table 2.

Table 1. Possible sets of pins' location -X coordinate.

px <sub>iwl</sub>	w = 1		w = 2		w = 3		w = 4	
	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2
i = 1	2	3	4	5	6	7	7	8
i = 2	3	4	8	7	6	7	5	4
i = 3	1	2	4	4	4	3	1	1
i = 4	2	3	4	5	6	7	7	8
i = 5	4	5	9	8	7	8	6	6

Table 2. Possible sets of pins' location –Y coordinate.

PY <sub>iwl</sub>	w = 1		w = 2		w = 3		w = 4	
	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2
i = 1	2	2	4	5	4	3	2	2
i = 2	8	8	8	8	5	6	5	6
i = 3	2	2	3	4	5	5	4	3
i = 4	2	2	4	5	4	3	2	2
i = 5	6	6	6	6	3	4	3	4

The parameters for the second and third problem are given in Appendix A. The results of running different models are summarized in table 3.

Table 3. Summary of findings.

	Number of parts	Basic model	Translation and rotation (nonlinear)
Number of fixed pins	5	2	12
	10	7	23
	50	29	100

The results show a significant improvement in the results by adding the translation and rotation flexibility to the model. It should be noted that based on model's non-linearity nature, the solution procedure becomes more complex and solution time is increased significantly (i.e. for the third problem, the solution time for basic and flexible models are 15 and 2500 minutes respectively).

**Conclusion**

Designing and fabricating fixtures is a significant portion of manufacturing costs which should be considered in lean manufacturing systems. By increasing products' varieties in today's competitive world, researchers will pay more attention to flexible fixture designs that can handle different workpieces with different geometries. In this paper, a hole-pattern modular fixture was used in a mid-volume, mid-variety production system in order to hold a variety of geometrically different products in a robotic assembly system. The modular cradle has a hole-pattern on its adapter plate for jiggling-pins to be inserted. Rearrangement of pins enables fixing different product geometries. However, the larger the number of changed pins, the more the productions' lead time increases. In this study, to minimize the total number of pins which have to be changed to fix different parts, an optimization model was developed to determine the best locations for placing different products on the cradle and the best locations for inserting pins to fix them. This model enables the system to take all the possible parts' translations and rotations into consideration. To evaluate the performance and to prove the efficacy of the proposed models, three different numerical examples with different sizes were solved. The results state that the model can significantly reduce the number of pins' replacements, which reduces the fixture's associated time and efforts.

**Future Work:**

As a direction for further research in this area, it is recommended to take the scheduling of different parts into consideration. This means that a robot in an assembly line can change the parts' order in order to find out the best possible pins' locations that minimize the number of pins' replacements.

**Appendix A**

*A.1. Model's parameters for 10 workpiece*

$$i = \{1,2, \dots, 10\} \quad w = \{1,2,3,4\} \quad l = \{1,2\}$$

Table 4. Possible sets of pins' location –X coordinate.

PX <sub>iwl</sub>	w = 1		w = 2		w = 3		w = 4	
	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2
i = 1	2	3	4	5	6	7	7	8
i = 2	3	4	8	7	6	7	5	4
i = 3	1	2	4	4	4	3	1	1
i = 4	3	4	5	6	7	8	8	9
i = 5	4	5	9	8	7	8	6	5
i = 6	2	3	5	5	5	4	2	2
i = 7	1	2	3	4	5	6	6	7
i = 8	2	3	7	6	5	6	4	3
i = 9	0	1	3	3	3	2	0	0
i = 10	2	3	4	5	6	7	7	8

Table 5. Possible sets of pins' location –Y coordinate.

PY <sub>iwl</sub>	w = 1		w = 2		w = 3		w = 4	
	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2
i = 1	2	2	4	5	4	3	2	2
i = 2	8	8	8	8	5	6	5	6
i = 3	2	2	3	4	5	5	4	3
i = 4	2	2	4	5	4	3	2	2
i = 5	8	8	8	8	5	6	5	6
i = 6	2	2	3	4	5	5	4	3
i = 7	2	2	4	5	4	3	2	2
i = 8	8	8	8	8	5	6	5	6
i = 9	2	2	3	4	5	5	4	3
i = 10	3	3	5	6	5	4	3	3

*A.2. Model's Parameters for 50 workpiece*

$$i = \{1,2, \dots, 10\} \quad w = \{1,2,3,4\} \quad l = \{1,2\}$$

Table 6. Possible sets of pins' location –X coordinate.

PX <sub>iwl</sub>	w = 1		w = 2		w = 3		w = 4	
	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2	l = 1	l = 2
i = 1	2	3	4	5	6	7	7	8
i = 2	3	4	8	7	6	7	5	4
i = 3	1	2	4	4	4	3	1	1
i = 4	3	4	5	6	7	8	8	9
i = 5	4	5	9	8	7	8	6	5
i = 6	2	3	5	5	5	4	2	2
i = 7	1	2	3	4	5	6	6	7
i = 8	2	3	7	6	5	6	4	3
i = 9	0	1	3	3	3	2	0	0
i = 10	2	3	4	5	6	7	7	8
i = 11	3	4	8	7	6	7	5	4
i = 12	1	2	4	4	4	3	1	1
i = 13	2	3	4	5	6	7	7	8
i = 14	3	4	8	7	6	7	5	4
i = 15	1	2	4	4	4	3	1	1

$i = 16$	3	4	5	6	7	8	8	9
$i = 17$	4	5	9	8	7	8	6	5
$i = 18$	2	3	5	5	5	4	2	2
$i = 19$	3	4	5	6	7	8	8	9
$i = 20$	4	5	9	8	7	8	6	5
$i = 21$	2	3	5	5	5	4	2	2
$i = 22$	1	2	3	4	5	6	6	7
$i = 23$	2	3	7	6	5	6	4	3
$i = 24$	0	1	3	3	3	2	0	0
$i = 25$	1	2	3	4	5	6	6	7
$i = 26$	2	3	7	6	5	6	4	3
$i = 27$	0	1	3	3	3	2	0	0
$i = 28$	2	2	4	5	4	3	2	2
$i = 29$	8	8	8	8	5	6	5	6
$i = 30$	2	2	3	4	5	5	4	3
$i = 31$	2	2	4	5	4	3	2	2
$i = 32$	8	8	8	8	5	6	5	6
$i = 33$	2	2	3	4	5	5	4	3
$i = 34$	2	2	4	5	4	3	2	2
$i = 35$	8	8	8	8	5	6	5	6
$i = 36$	2	2	3	4	5	5	4	3
$i = 37$	3	3	5	6	5	4	3	3
$i = 38$	9	9	9	9	6	7	6	7
$i = 39$	3	3	4	5	6	6	5	4
$i = 40$	1	1	3	4	3	2	1	1
$i = 41$	7	7	7	7	4	5	4	5
$i = 42$	1	1	2	3	4	4	3	2
$i = 43$	3	3	5	6	5	4	3	3
$i = 44$	9	9	9	9	6	7	6	7
$i = 45$	3	3	4	5	6	6	5	4
$i = 46$	1	1	3	4	3	2	1	1
$i = 47$	7	7	7	7	4	5	4	5
$i = 48$	1	1	2	3	4	4	3	2
$i = 49$	3	3	5	6	5	4	3	3
$i = 50$	9	9	9	9	6	7	6	7

Table 7. Possible sets of pins' location -Y coordinate.

$px_{iwt}$	$w = 1$		$w = 2$		$w = 3$		$w = 4$	
	$l = 1$	$l = 2$	$l = 1$	$l = 2$	$l = 1$	$l = 2$	$l = 1$	$l = 2$
$i = 1$	2	2	4	5	4	3	2	2
$i = 2$	8	8	8	8	5	6	5	6
$i = 3$	2	2	3	4	5	5	4	3
$i = 4$	2	2	4	5	4	3	2	2
$i = 5$	8	8	8	8	5	6	5	6
$i = 6$	2	2	3	4	5	5	4	3
$i = 7$	2	2	4	5	4	3	2	2
$i = 8$	8	8	8	8	5	6	5	6
$i = 9$	2	2	3	4	5	5	4	3
$i = 10$	3	3	5	6	5	4	3	3
$i = 11$	9	9	9	9	6	7	6	7
$i = 12$	3	3	4	5	6	6	5	4
$i = 13$	1	1	3	4	3	2	1	1
$i = 14$	7	7	7	7	4	5	4	5
$i = 15$	1	1	2	3	4	4	3	2
$i = 16$	3	3	5	6	5	4	3	3
$i = 17$	9	9	9	9	6	7	6	7
$i = 18$	3	3	4	5	6	6	5	4
$i = 19$	1	1	3	4	3	2	1	1
$i = 20$	7	7	7	7	4	5	4	5
$i = 21$	1	1	2	3	4	4	3	2
$i = 22$	3	3	5	6	5	4	3	3
$i = 23$	9	9	9	9	6	7	6	7
$i = 24$	3	3	4	5	6	6	5	4
$i = 25$	1	1	3	4	3	2	1	1
$i = 26$	7	7	7	7	4	5	4	5
$i = 27$	1	1	2	3	4	4	3	2
$i = 28$	2	3	4	5	6	7	7	8
$i = 29$	3	4	8	7	6	7	5	4
$i = 30$	1	2	4	4	4	3	1	1
$i = 31$	3	4	5	6	7	8	8	9
$i = 32$	4	5	9	8	7	8	6	5
$i = 33$	2	3	5	5	5	4	2	2
$i = 34$	1	2	3	4	5	6	6	7
$i = 35$	2	3	7	6	5	6	4	3

$i = 36$	0	1	3	3	3	2	0	0
$i = 37$	2	3	4	5	6	7	7	8
$i = 38$	3	4	8	7	6	7	5	4
$i = 39$	1	2	4	4	4	3	1	1
$i = 40$	2	3	4	5	6	7	7	8
$i = 41$	3	4	8	7	6	7	5	4
$i = 42$	1	2	4	4	4	3	1	1
$i = 43$	3	4	5	6	7	8	8	9
$i = 44$	4	5	9	8	7	8	6	5
$i = 45$	2	3	5	5	5	4	2	2
$i = 46$	3	4	5	6	7	8	8	9
$i = 47$	4	5	9	8	7	8	6	5
$i = 48$	2	3	5	5	5	4	2	2
$i = 49$	1	2	3	4	5	6	6	7
$i = 50$	2	3	7	6	5	6	4	3

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