Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 642 (2006) 9–12
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Standard Model extension with gravity and gravitational baryogenesis

Gaetano Lambiase [a,b]

[a] *Dipartimento di Fisica "E.R. Caianiello", Universitá di Salerno, 84081 Baronissi (Sa), Italy*
[b] *INFN, Gruppo Collegato di Salerno, Italy*

Received 4 September 2006; accepted 23 September 2006

Available online 2 October 2006

Editor: T. Yanagida

Abstract

The Standard Model extension with the inclusion of gravity is studied in the framework of the gravitational baryogenesis, a mechanism to generate the baryon asymmetry based on the coupling between the Ricci scalar curvature and the baryon current $(\partial_\mu R)J^\mu$. We show that, during the radiation era of the expanding universe, a non-vanishing time derivative of the Ricci curvature arises as a consequence of the coupling between the coefficients for the Lorentz and CPT violation and Ricci's tensor. The order of magnitude for these coefficients are derived from current bounds on baryon asymmetry.

PACS: 11.30.Cp; 11.30.Er; 98.80.Cq; 04.50.+h

Studies of a possible breakdown of the fundamental symmetries in physics have received in the last years a more and more growing interest and have been carried out in different areas (see for example [1–6]). Referring to Lorentz's and CPT symmetries, the more general setting in which they have been studied is the Standard Model Extension (SME) [1]. According to it, the violation of such fundamental symmetries follows from the observation that the vacuum solution of the theory could spontaneously violate the Lorentz and CPT invariance, even though them are preserved by the underlying theory. Modern tests for Lorentz and CPT invariance breakdown have been discussed in [7]. Recently, the SME has been extended to incorporate the gravitational interaction. Such studies have been formalized by Kostelecký and Bluhm in the papers [1,8].

This Letter concerns a cosmological aspect of the SME with gravity related to the so-called *gravitational* baryogenesis. The origin of the baryon number asymmetry is an open issue of the modern cosmology and particle physics. Measurements of CMB combined with the large structure of the universe [9], as well as predictions of big-bang nucleosynthesis [10] give indications that matter in the universe is dominant over antimatter. The order of magnitude of such a asymmetry is $\eta = \frac{n_B - n_{\bar{B}}}{s} \lesssim 9\times 10^{-11}$, where n_B ($n_{\bar{B}}$) is the baryon (antibaryon) number density, and s the entropy of the universe. Conventionally, it is argued that to generate (dynamically) the baryon asymmetry from an initial symmetric phase the following requirements are necessary [11]: (1) baryon number processes violating in particle interactions; (2) C and CP violation in order that processes generating B are more rapid with respect to $\bar{B}$; (3) out of the equilibrium: since $m_B = m_{\bar{B}}$, as follows from CPT symmetry, the equilibrium space phase density of particles and antiparticles are the same. To maintain the number of baryon and antibaryon different, i.e. $n_B \neq n_{\bar{B}}$, the reaction should freeze out before particles and antiparticles achieve the thermodynamical equilibrium. However, a dynamically violation of CPT allows to generate the baryon number asymmetry also in regime of thermal equilibrium [12]. A new mechanism to generate the baryon number asymmetry during the expansion of the universe has been proposed by Davoudiasl et al. [13]. In this mechanism the thermal equilibrium is maintained and CPT (and CP) symmetry is dynamically broken. The interaction is described by a coupling between the derivative of the Ricci scalar curvature R

E-mail address: lambiase@sa.infn.it (G. Lambiase).

0370-2693 © 2006 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2006.09.037

and the baryon current[1] J^μ

$$\frac{1}{M_*^2}\int d^4x\,\sqrt{-g}\,J^\mu\partial_\mu R, \tag{1}$$

where M_* is the cutoff scale characterizing the effective theory. The operator (1) may arise from supergravity theories from a higher-dimensional operator [14].

A net baryon asymmetry can be generated in thermal equilibrium provided that there exist interactions violating the baryon number B. Such an asymmetry gets frozen-in after the decoupling temperature[2] T_D. Since the scalar curvature only depends on cosmic time, the effective chemical potential for baryons following from Eq. (1) is $\mu_B=\dot R/M_*^2$ (for antibaryon one has $\mu_{\bar B}=-\mu_B$). In the regime $T\gg m_B$, the net baryon number density at the equilibrium is given by $n_B=g_b\mu_B T^2/6$. Here g_b represents the number of intrinsic degrees of freedom of baryons. The baryon number to entropy ratio evaluated at the decoupling, leads to [13]

$$\frac{n_B}{s}\simeq-\frac{15g_g}{4\pi^2 g_*}\frac{\dot R(t_D)}{M_*^2 T_D}, \tag{2}$$

where t_D is the decoupling time, $s=2\pi^2 g_{*s}T^3/45$, the dot stands for the derivative with respect to the cosmic time, and g_{*s} counts the total degrees of freedom for particles that contribute to the entropy of the universe. The latter assumes values close to the total degrees of freedom of effective massless particles g_* [15] ($g_{*s}\simeq g_*\sim 106$). In order that $n_B/s\neq 0$, a non-vanishing time derivative of the Ricci scalar is required. Einstein's field equations imply $R=-8\pi GT_g=-8\pi G(1-3w)\rho$, where T_g is trace of the energy–momentum tensor of matter $T_g^{\mu\nu}$, ρ is the matter density, $w=p/\rho$ with p the pressure. In the radiation dominated epoch of the standard Friedman–Robertson–Walker (FRW) cosmology, the constant w assumes the value (in the limit of exact conformal symmetry) $w=1/3$, so that the time derivative of the Ricci scalar is zero, as well as n_B/s. Differently to Einstein's theory of gravity, a net baryon asymmetry may be generated during the radiation era by Lorentz violating terms which couple to Ricci's and Riemann's tensors (for other scenarios see Refs. [13,16], and Ref. [17] for the case in which baryon current couples to scalar fields). Corrections induced by Lorentz and CPT violation also affects the baryon current J^μ in (1). Nevertheless the latter can be neglected since they give rise to corrections of the second order. We note that the effects on baryogenesis of spontaneous CPT violation in a string inspired scenario has been studied by Bertolami et al. [18] (see also [19]).

The SME with the inclusion of gravitational interactions [20] foresees that the effective action is $S=S_{\rm HE}+S_m+S_{\rm LV}$. $S_{\rm HE}=(16\pi G)^{-1}\int d^4x\,\sqrt{-g}(R-2\Lambda)$ is the Hilbert–Einstein action of general relativity (Λ is the cosmological constant), S_m the general matter action (which also includes Lorentz violating matter gravity coupling), and finally $S_{\rm LV}$ contains the leading Lorentz violating gravitational couplings

$$S_{\rm LV}=\frac{1}{16\pi G}\int d^4e\,\big(-uR+s^{\mu\nu}R_{\mu\nu}+t^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}\big). \tag{3}$$

The coefficients u, $s^{\mu\nu}$ and $t^{\kappa\lambda\mu\nu}$ are real and dimensionless. Moreover $s^{\mu\nu}$ and $t^{\kappa\lambda\mu\nu}$ inherit the Ricci and Riemann properties, respectively, and are traceless: $s^\mu{}_\mu=0$, $t^{\kappa\lambda}{}_{\kappa\lambda}=0$, $t^\kappa{}_{\mu\kappa\lambda}=0$. The PPN approximation of (3) has been studied in [21]. We restrict to the case $u=0$ and $t^{\kappa\lambda\mu\nu}=0$, therefore only the coefficients $s^{\mu\nu}$ control the Lorentz violation degrees of freedom. The variation of the action S with respect to the background metric yields the field equations [20]

$$G^{\mu\nu}-\big(T^{Rs}\big)^{\mu\nu}=8\pi GT_g^{\mu\nu}, \tag{4}$$

where $G^{\mu\nu}=R^{\mu\nu}-(R/2)g^{\mu\nu}$ is the standard Einstein tensor, and

$$\begin{aligned}\big(T^{Rs}\big)^{\mu\nu}={}&\frac12\big(s^{\alpha\beta}R_{\alpha\beta}-\nabla_\alpha\nabla_\beta\big)g^{\mu\nu}-s^{\mu\alpha}R_\alpha{}^\nu-s^{\nu\alpha}R_\alpha{}^\mu\\&+\frac12\big(\nabla_\alpha\nabla^\mu s^{\alpha\nu}+\nabla_\alpha\nabla^\nu s^{\alpha\mu}\big)-\frac12\nabla^2 s^{\mu\nu}.\end{aligned} \tag{5}$$

Tracing (5), one gets

$$R-\nabla_\alpha\nabla_\beta s^{\alpha\beta}=-8\pi GT_g, \tag{6}$$

where $T_g=g_{\mu\nu}T_g^{\mu\nu}$. Moreover, from Bianchi's identity $\nabla_\mu G^{\mu\nu}=0$, one obtains the relation

$$8\pi G\nabla_\mu T_g^\mu{}_\nu=-\frac12 R^{\alpha\beta}\nabla_\nu s_{\alpha\beta}+R^{\alpha\beta}\nabla_\beta s_{\alpha\nu}+\frac12 s_{\alpha\nu}\nabla^\alpha R. \tag{7}$$

The Friedman–Robertson–Walker (FRW) metric with zero spatial curvature ($k=0$) is

$$ds^2=dt^2-a^2(t)\big[dx^2+dy^2+dz^2\big]. \tag{8}$$

We assume that the coefficients $s^{\mu\nu}$ preserve the FRW symmetries (the universe is isotropic and homogeneous), so that $s^{ij}=s^{00}\delta^{ij}/3$. After lengthly calculations, the 00 and ij components of (4), and Eq. (7) read

$$3\frac{\dot a^2}{a^2}+\frac{\dot a}{a}\dot s^{00}-3\left(\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}\right)s^{00}=8\pi G\rho, \tag{9}$$

$$\frac{\ddot a}{a}+\frac{\dot a^2}{2a^2}+\left[\frac{\ddot s^{00}}{6}+\frac{\dot a}{a}\dot s^{00}+\left(\frac76\frac{\ddot a}{a}+\frac56\frac{\dot a^2}{a^2}\right)s^{00}\right]=-8\pi Gp, \tag{10}$$

$$\begin{aligned}8\pi G\left[\frac{1}{a^3}\frac{\partial}{\partial t}\big(\rho a^3\big)+3p\frac{\dot a}{a}\right]={}&-\left(2\frac{\ddot a}{a}+\frac{\dot a^2}{a^2}\right)\dot s^{00}\\&-\left[3\frac{\dddot a}{a}+7\frac{\ddot a}{a}\frac{\dot a}{a}+2\left(\frac{\dot a}{a}\right)^3\right]s^{00},\end{aligned} \tag{11}$$

where $\rho=T_g{}^0{}_0$ and $p=T_g{}^i{}_i$ (no sum over i). Eq. (6) reads

$$R=\ddot s^{00}+7\frac{\dot a}{a}\dot s^{00}+4\left(\frac{\ddot a}{a}+2\frac{\dot a^2}{a^2}\right)s^{00}-8\pi G(\rho-3p). \tag{12}$$

[1] Actually, the current J^μ can be replaced by any current yielding a net $B-L$ charge in thermal equilibrium (B and L are the baryon and lepton numbers, respectively), so that the asymmetry is not wiped out by the electroweak anomaly [24].

[2] It is assumed that the B-violating processes are generated by an operator of mass dimension $D=4+n$ [13], which decouple to T_D. The interaction rate is $\Gamma_B=T^{2n+1}/M_B^{2n}$, where M_B is the mass scale associated with the operator that generates B-violating processes.

Notice that the energy conservation (11) is fulfilled for ρ and p given by (9) and (10).

In the conventional general relativity, the pressure p and density ρ are related, during the radiation era, by the relation $p=\rho/3$ ($w=1/3$). From the Einstein field equations it follows that the scale factor depends on cosmic time as $a(t)\sim t^{1/2}$, the matter density as $\rho_R(t)=\frac{3}{32\pi G}t^{-2}\sim a^{-4}$, and finally the temperature as $T_R^2(t)=\frac{3}{4\pi}\sqrt{\frac{5}{Gg_*\pi}}t^{-1}\sim a^{-2}$. In the case of SME with gravity, the constant $w=1/3$ is corrected by coefficients breaking the Lorentz symmetry, and therefore may vary with the time t.

As discussed by Colladay and McDonald [22], the formalism of statistical mechanics (as well as the laws of thermodynamics) in presence of Lorentz breakdown is the same as for conventional statistical mechanics. We therefore use the standard definition for the density ρ and the pressure p. The general expressions for them are

$$\rho=g\int\frac{d^3P}{(2\pi)^3}\frac{u_\mu u_\nu P^\mu P^\nu}{P^0}f, \tag{13}$$

$$p=\frac{g}{3}\int\frac{d^3P}{(2\pi)^3}\frac{(u_\mu u_\nu-g_{\mu\nu})P^\mu P^\nu}{P^0}f, \tag{14}$$

with $u^\mu=(1,0,0,0)$ the four-velocity of the fluid, $f=(e^{E/T}\pm 1)$ is the Fermi–Dirac/Bose–Einstein distribution. The dispersion relation reads $g_{\mu\nu}P^\mu P^\nu=m^2+2c_{\mu\nu}P^\mu P^\nu+\cdots$, where the ellipsis represents other contributions of SME. $c_{\mu\nu}$ in the SME with gravity may depend on the position. In the radiation era, the universe was filled by photons and ultra-relativistic fermion particles (baryons, electrons, neutrinos). According to SME [1], let us assume the photon sector as conventional, so that coefficients for the Lorentz violation enter in the dispersion relations of ultra-relativistic particles. Without loss of generality, let us consider Lorentz violating corrections only for the baryon sector, i.e. $c_{00}^{(b)}\neq 0$ (otherwise, $c_{\mu\nu}^{(b)}$ is replaced by $\sum_a q_a c_{\mu\nu}^{(a)}$, where $a=e,\nu,b,\ldots$, and q_a are constants which accounts for the statistics of particles). Requiring the FRW symmetry, the non-vanishing (traceless) coefficients $c_{\mu\nu}(\ll 1)$ for the Lorentz violation are the diagonal components, with $c^{ij}=c^{00}\delta^{ij}/3$ (see [23]). As a consequence, one gets $c_{\mu\nu}P^\mu P^\mu\simeq 4c_{00}E^2/3$, $E\simeq(1-4c_{00}/3)P$, and the massive term turns out to be modified as $m^2(1+c^{00})$. The latter may be neglected for ultra-relativistic particles ($P^2\ggg m^2$). Eq. (14) then reads

$$p=\frac{\rho}{3}\left(1-\frac{7}{3}\frac{g_b}{g_*}c_{00}^{(b)}\right). \tag{15}$$

To account for corrections induced by c^{00} and s^{00} coefficients, also the scale factor turns out to be modified, and therefore we set $a\to a(1+\delta)$, with $\delta\ll 1$. A solution of Eqs. (9)–(11) can be derived with the ansatz

$$a\simeq t^\alpha,\qquad s^{00}=St^\gamma,\qquad \delta=Dt^\gamma,\qquad c_{00}^{(b)}=Ct^\gamma.$$

To leading order in s^{00}, $c_{00}^{(b)}$, and δ, and using (15), Eqs. (9)–(11) admit the following solution:

$$\gamma=-\frac{3}{2},\qquad \alpha=\frac{1}{2}, \tag{16}$$

$$\rho=\rho_R+\frac{8\gamma S+\frac{21}{2}\frac{g_b}{g_*}C}{32\pi G(3\gamma+1)}\frac{1}{t^{2-\gamma}}, \tag{17}$$

$$p=\frac{\rho_R}{3}+\frac{8\gamma S+\frac{7(1-6\gamma)}{2}\frac{g_b}{g_*}C}{96\pi G(3\gamma+1)}\frac{1}{t^{2-\gamma}}, \tag{18}$$

$$D=\frac{-1}{2\gamma(\gamma+1)}\left[(\gamma+2)\left(\gamma+\frac{1}{2}\right)S+\frac{7}{4}\frac{g_b}{g_*}C\right]. \tag{19}$$

The constants S and C are free and their combination is fixed by the observed baryon number asymmetry. Moreover, Eqs. (17) and (18) indicate that corrections induced by Lorentz violating terms fall down faster than $\rho_R\sim 1/t^2$.

The interaction (1) generates a net baryon asymmetry provided $\dot{R}\neq 0$. s^{00} and $c_{00}^{(b)}$-corrections prevent indeed the Ricci curvature to vanish, as well as its first time derivative. From (12) it follows in fact

$$\dot{R}=-\frac{(1-\gamma/2)\Pi}{t^{3-\gamma}}, \tag{20}$$

where

$$\Pi\equiv\left|S+\frac{7g_b}{2g_*}C\right|.$$

The temperature is derived by using Eqs. (13) and (17), and is given by $T^4(t)=T_R^4(t)+\delta_T/t^{2-\gamma}$, where δ_T includes the C and S-corrections. In computing n_B/s (Eq. (2)), δ_T may be neglected since gives rise to corrections of the second order. The net baryon asymmetry at the decoupling temperature T_D is hence

$$\frac{n_B}{s}\simeq 1.1\times 10^5\Pi\ \mathrm{GeV}^{-\gamma}\left(\frac{T_D}{m_{\mathrm{Pl}}}\right)^{5-\gamma}\left(\frac{T_D}{\mathrm{GeV}}\right)^{-\gamma}\left(\frac{m_{\mathrm{Pl}}}{M_*}\right)^2, \tag{21}$$

where $m_{\mathrm{Pl}}\sim 10^{19}$ GeV is the Planck mass. As pointed out in [13], a possible choice of the cutoff scale is $M_*=m_{\mathrm{Pl}}$ if $T_D=M_I$, where $M_I\sim 3.3\times 10^{16}$ GeV is the upper bound on the tensor mode fluctuation constraints in inflationary scale. This choice is particular interesting because implies that tensor mode fluctuations should be observed in the next generation of experiments. Since the constraint of the observed baryon asymmetry is $n_B/s\lesssim 9\times 10^{-11}$, one gets

$$\Pi\lesssim 5.8\times 10^{-24.5}\ \mathrm{GeV}^{-3/2}\sim 3.1\times 10^{-60.5}\ \mathrm{s}^{3/2}. \tag{22}$$

An upper bound for s^{00} is therefore

$$s^{00}\lesssim 7.6\times 10^4\left(\frac{t_{\mathrm{Pl}}}{t}\right)^{3/2}\quad \text{for } t\gtrsim 10^{-38}\ \mathrm{s}, \tag{23}$$

where $t_{\mathrm{Pl}}\sim 5.4\times 10^{-44}$ s is the Planck time. Similarly for $c_{00}^{(b)}$:

$$c_{00}^{(b)}\lesssim 1.1\times 10^6\left(\frac{t_{\mathrm{Pl}}}{t}\right)^{3/2}\quad \text{for } t\gtrsim 10^{-38}\ \mathrm{s}. \tag{24}$$

As we can see, coefficients leading to deviations from general relativity, as well as to Lorentz's and CPT violation, decrease during the expanding phase of the universe, and therefore corrections become more and more negligible. Of course such results refer to radiation dominated era. Different epochs, such as, for example, matter dominated era, lead to different field equations, hence to a different time-dependence of the Lorentz and

CPT violating coefficients, with potentially interesting consequences on cosmological scenarios offered by the SME with gravity.

In conclusion, a cosmological consequence of the gravitational sector in the SME has been investigated in this Letter. Such a study is related to the (CPT violating) interaction between the Ricci scalar curvature and the baryon current (Eq. (1)). During the phase of the expanding universe dominated by radiation, SME with the inclusion of gravity provides a framework in which the baryon asymmetry may be gravitationally induced. The current estimations on the observed baryon asymmetry yield an order of magnitude of coefficients breaking the Lorentz and CPT symmetry.

Acknowledgements

It is a pleasure to thank V. Alan Kostelecký for comments and suggestions.

References

[1] D. Colladay, V.A. Kostelecký, Phys. Rev. D 58 (1998) 116002;
D. Colladay, V.A. Kostelecký, Phys. Rev. D 55 (1997) 6760.
[2] J. Ellis, K. Farakos, N.E. Mavromatos, V.A. Mitsou, D.V. Nanopoulos, Astrophys. J. 535 (2000) 139;
J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Phys. Rev. D 61 (2000) 027503.
[3] J. Alfaro, H.A. Morales-Téctol, L.F. Urrutia, Phys. Rev. Lett. 84 (2000) 2318;
J. Alfaro, H.A. Morales-Téctol, L.F. Urrutia, Phys. Rev. D 65 (2002) 103509;
J. Alfaro, H.A. Morales-Téctol, L.F. Urrutia, Phys. Rev. D 66 (2002) 124006;
R. Gambini, J. Pullin, Phys. Rev. D 59 (1999) 124021;
A. Ashtekar, Lectures on New Perturbative Canonical Gravity, World Scientific, 1991;
C. Rovelli, S. Speziale, Phys. Rev. D 67 (2003) 064019.
[4] G. Amelino-Camelia, Int. J. Mod. Phys. D 11 (2002) 35;
S.M. Carroll, J. Harvey, V.A. Kostelecký, C.D. Lane, T. Okamoto, Phys. Rev. Lett. 87 (2001) 141601.
[5] V.A. Kostelecký, R. Lehnert, M.J. Perry, Phys. Rev. D 68 (2003) 123511;
O. Bertolami, R. Lehnert, R. Potting, A. Ribeiro, Phys. Rev. D 69 (2004) 083513.
[6] T. Jacobson, D. Mattingly, Phys. Rev. D 64 (2001) 024028;
T. Jacobson, D. Mattingly, Phys. Rev. D 70 (2004) 024003.
[7] D. Mattingly, Living Rev. Relativ. 8 (2005) 5;
H. Vucetich, gr-qc/0502093;
R. Bluhm, hep-ph/0506054.
[8] R. Bluhm, V.A. Kostelecký, Phys. Rev. D 71 (2005) 065008.
[9] C.L. Bennett, et al., Astrophys. J. Suppl. Ser. 148 (2003) 15.
[10] S. Burles, K.M. Nollet, M.S. Turner, Phys. Rev. D 63 (2001) 063512.
[11] A.D. Sakharov, JETP Lett. 5 (1967) 24.
[12] A. Cohen, D. Kaplan, Phys. Lett. B 199 (1987) 251.
[13] H. Davoudiasl, R. Kitano, G.D. Kribis, H. Murayama, P. Steinhardt, Phys. Rev. Lett. 93 (2004) 201301.
[14] T. Kugo, S. Uehara, Nucl. Phys. B 222 (1983) 125;
T. Kugo, S. Uehara, Prog. Theor. Phys. 73 (1985) 235.
[15] E. W Kolb, M.S. Turner, The Early Universe, Addison–Wesley, 1989.
[16] M.C. Bento, R. Gonzales Felipe, N.M.C. Santos, Phys. Rev. D 71 (2005) 123517;
H. Li, M. Li, X. Zhang, Phys. Rev. D 70 (2004) 047302;
B. Feng, H. Li, M. Li, X. Zhang, Phys. Lett. B 620 (2005) 27;
T. Shiromizu, K. Koyama, JCAP 07 (2004) 011.
[17] C.-Y. Cheng, Y.G. Shen, B. Feng, hep-ph/0508059;
A. De Felice, M. Trodden, Phys. Rev. D 72 (2005) 043512;
M. Li, W.B. Feng, X. Zhang, Phys. Rev. D 65 (2002) 103511;
M. Li, X. Zhang, Phys. Lett. B 573 (2003) 20;
G.L. Alberghi, R. Casadio, A. Tronconi, hep-ph/0310052.
[18] O. Bertolami, D. Colladay, A.V. Kostelecký, R. Potting, Phys. Lett. B 395 (1997) 178.
[19] S.M. Carroll, J. Shu, hep-ph/050081.
[20] V.A. Kostelecký, Phys. Rev. D 69 (2004) 105009.
[21] Q.G. Bailey, V.A. Kostelecký, gr-qc/0603030.
[22] D. Colladay, P. McDonald, Phys. Rev. D 70 (2004) 125007.
[23] G. Lambiase, Phys. Rev. D 72 (2005) 087702;
D. Colladay, V.A. Kostelecký, Phys. Lett. B 511 (2001) 209.
[24] V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.