



Running cosmological constant with observational tests



Chao-Qiang Geng^{a,b,c,*}, Chung-Chi Lee^b, Kaituo Zhang^{d,b}

^a Chongqing University of Posts & Telecommunications, Chongqing, 400065, China

^b National Center for Theoretical Sciences, Hsinchu, 300, Taiwan

^c Department of Physics, National Tsing Hua University, Hsinchu, 300, Taiwan

^d Anhui Normal University, Wuhu, Anhui, 24100, China

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ABSTRACT

We investigate the running cosmological constant model with dark energy linearly proportional to the Hubble parameter, $\Lambda = \sigma H + \Lambda_0$, in which the Λ CDM limit is recovered by taking $\sigma = 0$. We derive the linear perturbation equations of gravity under the Friedmann–Lemaître–Robertson–Walker cosmology, and show the power spectra of the CMB temperature and matter density distribution. By using the Markov chain Monte Carlo method, we fit the model to the current observational data and find that $\sigma H_0/\Lambda_0 \lesssim 2.63 \times 10^{-2}$ and 6.74×10^{-2} for $\Lambda(t)$ coupled to matter and radiation-matter, respectively, along with constraints on other cosmological parameters.

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1. Introduction

The type-Ia supernova observations [1,2] have shown that our universe is undergoing a late-time accelerating expansion, which is caused by Dark Energy [3]. The simplest way to realize such a late-time accelerating mechanism is to introduce a cosmological constant to the gravitational theory, such as that in the Λ CDM model. This model fits current cosmological observations very well, but there exist several difficulties, such as the “fine-tuning” [4,5] and “coincidence” [6] problems.

In this work, we will concentrate on the latter problem [7], which has been extensively explored in the literature. One of the popular attempts is the running Λ model, in which the cosmological constant evolves in time and decays to matter in the evolution of the universe [8–20], so that the present energy densities of dark energy and dark matter are of the same order of magnitude. Its observational applications have been investigated in Refs. [21–23]. In our study, we are interested in the specific model with $\Lambda = \sigma H$ [24–29], which would originate from the theory with the QCD vacuum condensation associated with the chiral

phase transition [30–34]. In this scenario, the cosmological constant decays to matter (non-relativistic) and radiation (relativistic), leading to a large number of particles created in the cosmological evolution. Without loss of generality, we phenomenologically extend this model to include that Λ additionally couples to radiation with $\Lambda = \sigma H + \Lambda_0$ [35–37], in which the Λ CDM limit can be realized if $\sigma = 0$. In this scenario, when dark energy dominates the universe, the decay rate of Λ is reduced, and the late-time accelerating phase occurs, describing perfectly the evolution history of the universe. As a result, it is reasonable to go further to analyze the cosmological behavior of this model at the sub-horizon scale.

In this paper, we examine the matter power spectrum $P(k)$ and CMB temperature perturbations in the linear perturbation theory of gravity. By using the Markov chain Monte Carlo (MCMC) method, we perform the global fit from the current observational data and constrain the model.

This paper is organized as follows: In Sec. 2, we introduce the $\Lambda(t)$ CDM model and review its background cosmological evolutions. In Sec. 3, we calculate the linear perturbation theory and illustrate the power spectra of the matter distribution and CMB temperature by the **CAMB** program [38]. In Sec. 4, we use the **CosmoMC** package [39] to fit the model from the observational data and show the constraints on cosmological parameters. Our conclusions are presented in Sec. 5.

* Corresponding author.

E-mail addresses: geng@phys.nthu.edu.tw (C.-Q. Geng), chungchi@mx.nthu.edu.tw (C.-C. Lee), ktzhang@ahnu.edu.cn (K. Zhang).

2. The running cosmological constant model

We start with the Einstein equation, given by

$$R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R + \Lambda(t)g_{\mu\nu} = \kappa^2 T_{\mu\nu}^M, \quad (1)$$

where $\kappa^2 = 8\pi G$, $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar, $\Lambda(t)$ is the time-dependent cosmological constant, and $T_{\mu\nu}^M$ is the energy-momentum tensor of matter and radiation. In the Friedmann-Lemaître-Robertson-Walker (FLRW) case,

$$ds^2 = a^2(\tau) \left[-d\tau^2 + \delta_{ij}dx^i dx^j \right], \quad (2)$$

we obtain

$$H^2 = \frac{a^2 \kappa^2}{3} (\rho_M + \rho_\Lambda), \quad (3)$$

$$\dot{H} = -\frac{a^2 \kappa^2}{2} (\rho_M + P_M + \rho_\Lambda + P_\Lambda), \quad (4)$$

where τ is the conformal time, $H = da/(a d\tau)$ represents the Hubble parameter, ρ_M (P_M) corresponds to the energy density (pressure) of matter and radiation, and ρ_Λ (P_Λ) is the energy density (pressure) of the cosmological constant. We note that from the relation of $\rho_\Lambda = -P_\Lambda = \kappa^{-2}\Lambda(t)$, derived from Eq. (1), one has the equation of state (EoS) of Λ to be

$$w_\Lambda \equiv \frac{P_\Lambda}{\rho_\Lambda} = -1. \quad (5)$$

In Eq. (1), we consider $\Lambda(t)$ to be a linear function of the Hubble parameter, given by [29,35–37]

$$\Lambda = \sigma H + \Lambda_0, \quad (6)$$

where σ and Λ_0 are two free parameters. From Eq. (6), we can write ρ_Λ with two dimensionless parameters $\lambda_{0,1}$ as

$$\rho_\Lambda = \rho_\Lambda^0 \left[\lambda_0 + \lambda_1 \left(\frac{H}{H_0} \right) \right], \quad (7)$$

where $\rho_\Lambda^0 \equiv \rho_\Lambda|_{z=0}$ is the current dark energy density with the condition $\lambda_0 + \lambda_1 = 1$ and $\lambda_1 = \sigma H_0 / (\sigma H_0 + \Lambda_0)$. Note that λ_0 has been treated as a constant of integration and set to zero in Ref. [29]. Without loss of generality, we will keep λ_0 as a free parameter with the Λ CDM model recovered when $\lambda_0 \rightarrow 1$.

Substituting Eq. (7) into the conservation equation $\nabla^\mu (T_{\mu\nu}^M + T_{\mu\nu}^\Lambda) = 0$, we have

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = \dot{\rho}_\Lambda \propto \dot{H} \neq 0, \quad (8)$$

resulting in that dark energy unavoidably couples to matter and radiation, given by

$$\dot{\rho}_m + 3H\rho_m = Q_m, \quad (9)$$

$$\dot{\rho}_r + 4H\rho_r = Q_r, \quad (10)$$

where $Q_{m,r}$ are the decay rate from $\Lambda(t)$ to matter and radiation, taken to be

$$Q_i = \frac{\dot{\rho}_\Lambda C_i (\rho_i + P_i)}{\sum_{j=m,r} C_j (\rho_j + P_j)}, \quad (11)$$

respectively. Note that the analytical solution of Eq. (8) has been obtained with $\lambda_0 = 0$ and $w_M = \text{constant}$ in Refs. [24,25]. However, if $\lambda_0 \neq 0$ and $\rho_M = \rho_m + \rho_r$, composited of multi-fluid with EoS $w_r \neq w_m$, the analytical solution no longer exists.

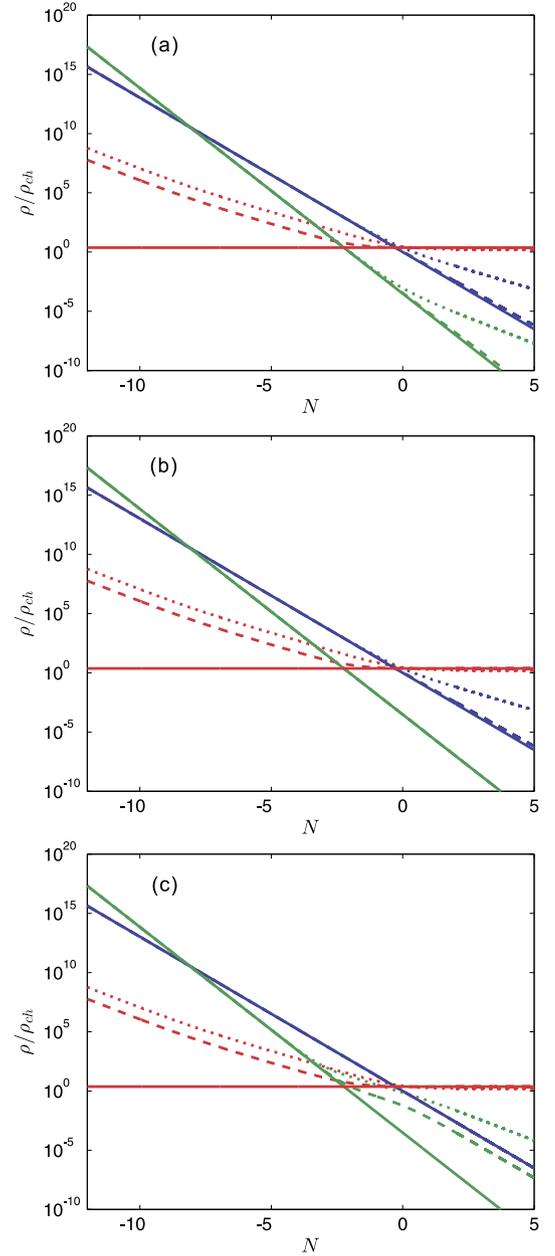


Fig. 1. Evolutions of ρ_m (blue line), ρ_r (green line) and ρ_Λ (red line) with (a), (b) and (c) corresponding to $(C_r, C_m) = (1, 1)$, $(0, 1)$ and $(1, 0)$, where the solid, dashed and dotted lines represent $(\lambda_0, \lambda_1) = (1, 0)$, $(0.9, 0.1)$ and $(0, 1)$, respectively. The initial conditions are taken as $\rho_m a^3 / \rho_{ch} = 1$ and $\rho_r a^4 / \rho_{ch} = 3 \times 10^{-4}$ at $N \equiv \ln a = -12$, where ρ_{ch} is the characteristic energy density. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

In Fig. 1, we show the cosmological evolutions of ρ_m (blue line), ρ_r (green line) and ρ_Λ (red line), normalized by the characteristic energy density ρ_{ch} , as functions of the e-folding $N \equiv \ln a$ with $\rho_m a^3 / \rho_{ch} = 1$ and $\rho_r a^4 / \rho_{ch} = 3 \times 10^{-4}$ at $N = -12$, where (C_r, C_m) are (a) $(1, 1)$, (b) $(1, 0)$ and (c) $(0, 1)$ with $(\lambda_0, \lambda_1) = (1, 0)$, $(0.9, 0.1)$ and $(0, 1)$, corresponding to the solid, dashed and dotted lines, respectively. In Fig. 1c, we observe that if dark energy fully decays to radiation, ρ_r can be of the same order of ρ_m at $\lambda_1 \gtrsim 0.1$, which violates the current observations. On the contrary, this problem never occurs if $\Lambda(t)$ only couples to matter as shown in Fig. 1b. This behavior allows us to fix $C_m = 1$ and keep C_r to be a free parameter in the later study. In Fig. 2, we present $a^3 \rho_m$ (blue

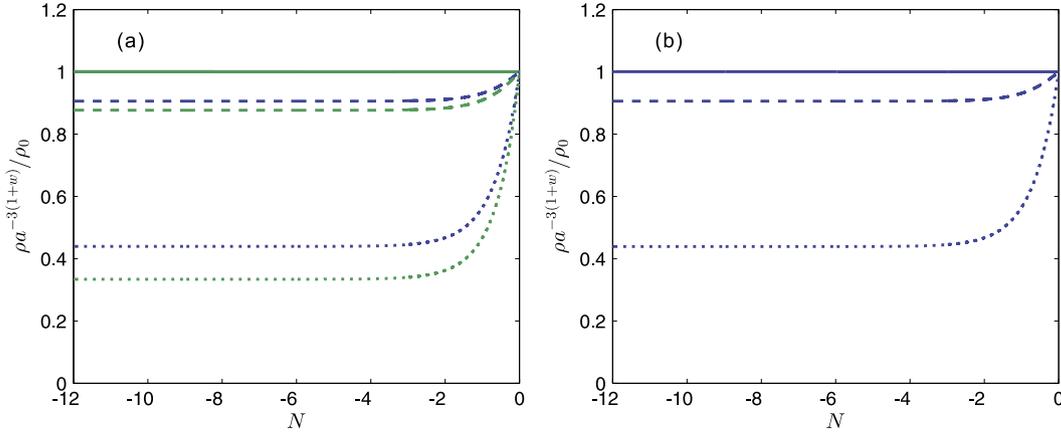


Fig. 2. Evolutions of $\rho_m a^3$ (blue lines) and $\rho_r a^4$ (green lines) with (a) $C_r = 1$ and (b) $C_r = 0$, where legend is the same as in Fig. 1. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

line) and $a^4 \rho_r$ (green line) as functions of N with $(\lambda_0, \lambda_1) = (1, 0)$, $(0.9, 0.1)$ and $(0, 1)$, where the boundary conditions are the same as those in Fig. 1. From the plots in Figs. 1 and 2, one can see that the cosmological constant decouples to matter and radiation (corresponding to $\rho_r a^{-3(1+w_i)} = \text{constant}$) when $(\lambda_0, \lambda_1) \rightarrow (1, 0)$. On the other hand, a large number of matter and radiation are created by the decay of dark energy at the late-time of the universe with $(\lambda_0, \lambda_1) \rightarrow (0, 1)$. When dark energy dominates the universe, ρ_Λ decays slowly and our universe turns into the accelerating expansion phase. Clearly, this scenario is suitable to describe the dark energy problem no matter what values for λ_0 and λ_1 are used. To examine this model in detail, we need to study the linear perturbation theory of gravity and investigate the effects at the subhorizon scale.

3. Cosmological perturbation in running cosmological constant model

We focus on the perturbation theory with the synchronous gauge and explore the power spectra of matter and the CMB temperature in the running cosmological constant model. Under the FLRW background, the metric perturbation is given by [40]

$$ds^2 = a^2(\tau) \left[-d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right], \quad (12)$$

where

$$h_{ij} = \int d^3 k e^{i\vec{k}\cdot\vec{x}} \left[\hat{k}_i \hat{k}_j h(\vec{k}, \tau) + 6 \left(\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) \eta(\vec{k}, \tau) \right], \quad (13)$$

$i, j = 1, 2, 3$, h and η are two scalar perturbations in the synchronous gauge, and $\hat{k} = \vec{k}/k$ is the k -space unit vector. The matter (baryon, cold dark matter and massive neutrino) and radiation (photon and massless neutrino) density perturbations can be derived from the conservation equation $\nabla^\mu (T_{\mu\nu}^M + T_{\mu\nu}^\Lambda) = 0$ with $\delta T_0^0 = \delta \rho_M$, $\delta T_i^0 = -T_0^i = (\rho_M + P_M) v_M^i$ and $\delta T_j^i = \delta P_M \delta_j^i$, given by [27,41],

$$\delta_M = -(1 + w_M) \left(\theta_M + \frac{\dot{h}}{2} \right) - 3H \left(\frac{\delta P_M}{\delta \rho_M} - w_M \right) \delta_M - \frac{Q_M}{\rho_M} \delta_M, \quad (14)$$

where $\delta_M \equiv \delta \rho_M / \rho_M$, $\theta_M = ik_i v_M^i$, and Q_M is the decay rate in Eq. (11).

To show how the running cosmological constant scenario influences the matter density perturbation and CMB temperature

fluctuation, we perform the open-source program **CAMB** [38] with the model in Eq. (7), and modify the evolution of the density perturbation in Eq. (14) as well as the background density evolutions $\rho_\Lambda(z)$, $\rho_m(z)$ and $\rho_r(z)$, solved from Eqs. (8)–(10), respectively. Since the dark energy density ratio, $\rho_\Lambda / \rho_c \propto H^{-1}$, is negligible in the early universe, the running cosmological constant model shares the same initial condition as the Λ CDM model with $\rho_M = \rho_M|_{a \rightarrow 0}$. Besides, the matter-radiation equality z_{eq} also changes in this model, given by

$$\frac{\rho_m(Z = z_{eq})}{\rho_r(Z = z_{eq})} = 1. \quad (15)$$

In Fig. 3, we present the matter power spectrum $P(k)$ as a function of the wavenumber k with (a) $C_r = 1$ and (b) $C_r = 0$, along with $\Omega_b h^2 = 2.23 \times 10^{-2}$, $\Omega_c h^2 = 0.119$, $\Sigma m_\nu = 0.06$ eV and $(\lambda_0, \lambda_1) = (1, 0)$ (black line), $(0.9, 0.1)$ (red line), $(0.5, 0.5)$ (blue line) and $(0, 1)$ (cyan line), respectively. Obviously, the matter power spectrum $P(k)$ is suppressed at a large k , and the deviation from Λ CDM (black line) is significant for $\lambda_1 \gg 0$. As a result, the allowed parameter space for λ_1 can be estimated to be small. The suppression of $P(k)$ can be realized from Eq. (14) with $Q_m \propto -\dot{H} \geq 0$. As λ_1 increases, the running cosmological model deviates from the Λ CDM limit, while the matter creation gets enhanced due to the dark energy decay as seen in Eq. (9). In addition, since dark energy is considered to be homogeneous and isotropic, $\delta \rho_\Lambda = 0$, so that the creation of matter smoothly distributes to our universe. Consequently, the density perturbation is diluted and the matter power spectrum is suppressed by the decay of Λ .

Fig. 4 depicts the CMB temperature power spectrum in the running cosmological constant model with (a) $C_r = 1$ and (b) $C_r = 0$, while the boundary conditions are the same as those in Fig. 3. One finds that the CMB temperature spectrum in $\Lambda(t)$ CDM significantly differs from that in Λ CDM (black line) when $\lambda_1 \gg 0$. This gives us a hint that λ_1 would be relatively small in order to fit the spectrum. Our results demonstrate that the allowed matter creation from the cosmological constant should be tiny.

4. Observational constraints

We now examine the possible ranges for λ_0 and λ_1 by the cosmological observations. We use the open-source **CosmoMC** program [39] with the MCMC method to perform the global fitting for the $\Lambda(t)$ CDM model. The dataset includes (i) *Planck* 2015 TT, TE, EE, low- l polarization and lensing from SMICA [42–44]; (ii) baryon acoustic oscillation (BAO) data from 6dF Galaxy Survey [45], SDSS

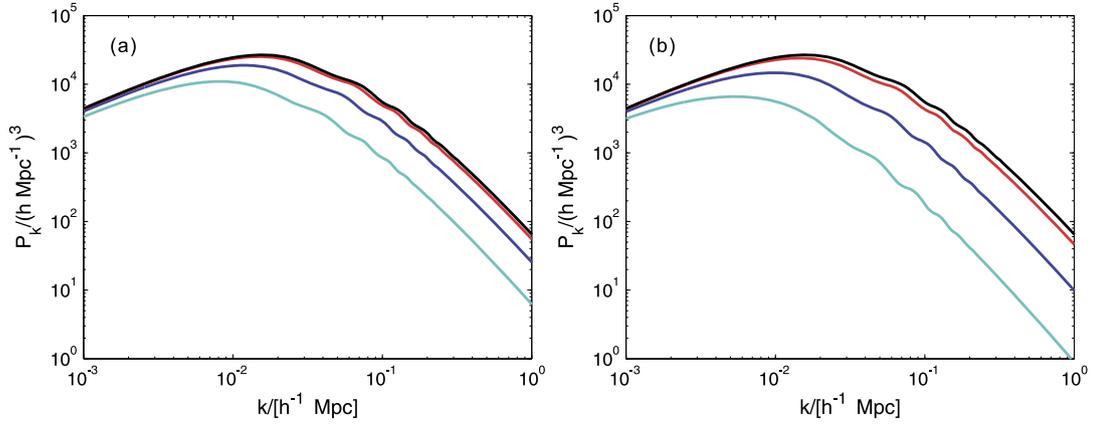


Fig. 3. The matter power spectrum $P(k)$ as a function of the wavelength $k = 2\pi/\lambda$ with (a) $C_r = 1$ and (b) $C_r = 0$, where $(\lambda_0, \lambda_1) = (1, 0)$ (black line), $(0.9, 0.1)$ (red line), $(0.5, 0.5)$ (blue line), $(0, 1)$ (cyan line), and the boundary conditions are taken to be $\Omega_b h^2 = 2.23 \times 10^{-2}$, $\Omega_c h^2 = 0.119$, $h = 0.7$ and $\Sigma m_\nu = 0.06$ eV, respectively. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

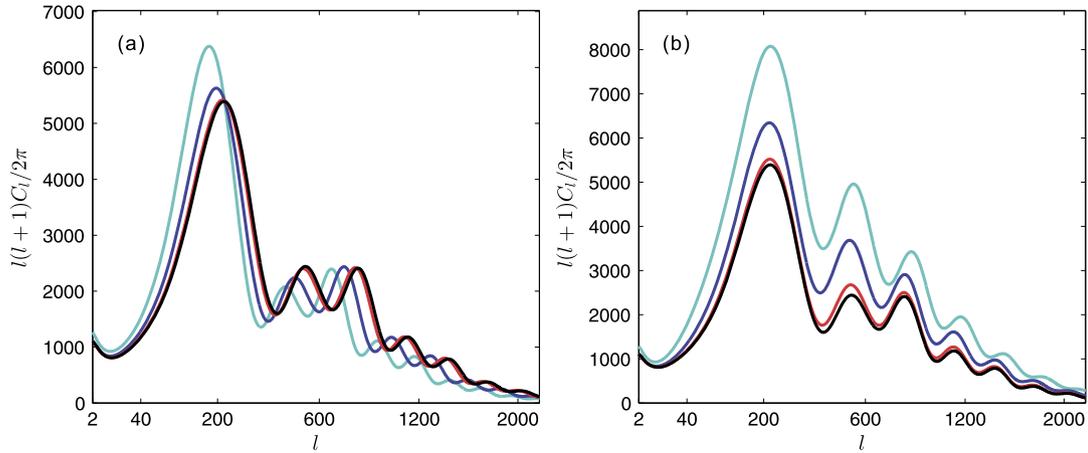


Fig. 4. The CMB temperature power spectrum for (a) $C_r = 1$ and (b) $C_r = 0$ with $T = 2.73$ K, where legend is the same as in Fig. 3. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Table 1

Priors for cosmological parameters with $\rho_\Lambda = \rho_\Lambda^0 [\lambda_0 + \lambda_1 H/H_0]$ and $\lambda_0 + \lambda_1 = 1$.

Parameter	Prior
Baryon density	$0.5 \leq 100\Omega_b h^2 \leq 10$
CDM density	$10^{-3} \leq \Omega_c h^2 \leq 0.99$
Optical depth	$0.01 \leq \tau \leq 0.8$
Neutrino mass sum	$0 \leq \Sigma m_\nu \leq 2$ eV
Model parameter λ_1	$0 \leq \lambda_1 \leq 0.3$

DR7 [46] and BOSS [47]; (iii) matter power spectrum data from SDSS DR4 and WiggleZ [48], and (iv) weak lensing data from CFHTLenS [49]. With this dataset, we keep the model parameter $\lambda_1 \geq 0$ to avoid the negative dark energy density and explore the constraints on λ_1 and other cosmological parameters. The priors of the various parameters are listed in Table 1, and our results are shown in Fig. 5.

From Fig. 5, we find that λ_1 is strongly constrained with $\lambda_1 = \sigma H_0 / (\sigma H_0 + \Lambda_0) \simeq \sigma H_0 / \Lambda_0 \lesssim 6.68 \times 10^{-2}$ (2.63×10^{-2}) for $C_r = 1(0)$ at 2σ -confidence level, which is consistent with the prediction in Sec. 3. Additionally, the best-fit $\chi_{\Lambda(t)}^2 = 13546.5$ (13545.7) for $C_r = 1(0)$ in the $\Lambda(t)$ CDM is slightly larger than $\chi_{\Lambda}^2 = 13545.3$ in the Λ CDM model. Since the cosmological observations prefer the Λ CDM limit, the effect from the decaying dark energy model is strongly confined. Finally, the allowed ranges for the various cos-

mological parameters with 2σ -confidence level are summarized in Table 2.

5. Conclusions

We have investigated the $\Lambda(t)$ CDM model with the dark energy decaying to both matter and radiation, in which $\Lambda(t) = \sigma H + \Lambda_0$. Although this scenario is suitable to describe the late-time accelerating universe at the background level, the linear perturbation analyses of the matter power and CMB temperature spectra have set a strong constraint on the model parameter λ_1 in Eq. (7). Explicitly, by performing the global fit from the observational data, we have obtained that $\lambda_1 \simeq \sigma H_0 / \Lambda_0 \lesssim 6.68 \times 10^{-2}$ (2.63×10^{-2}) and $\chi_{\Lambda(t)\text{CDM}}^2 = 13546.5(13545.7) \gtrsim \chi_{\Lambda\text{CDM}}^2 = 13545.3$ for $C_r = 1(0)$, implying that the current data prefers the Λ CDM limit. Constraints on other cosmological parameters in both $\Lambda(t)$ CDM and Λ CDM models have been also given in Table 2.

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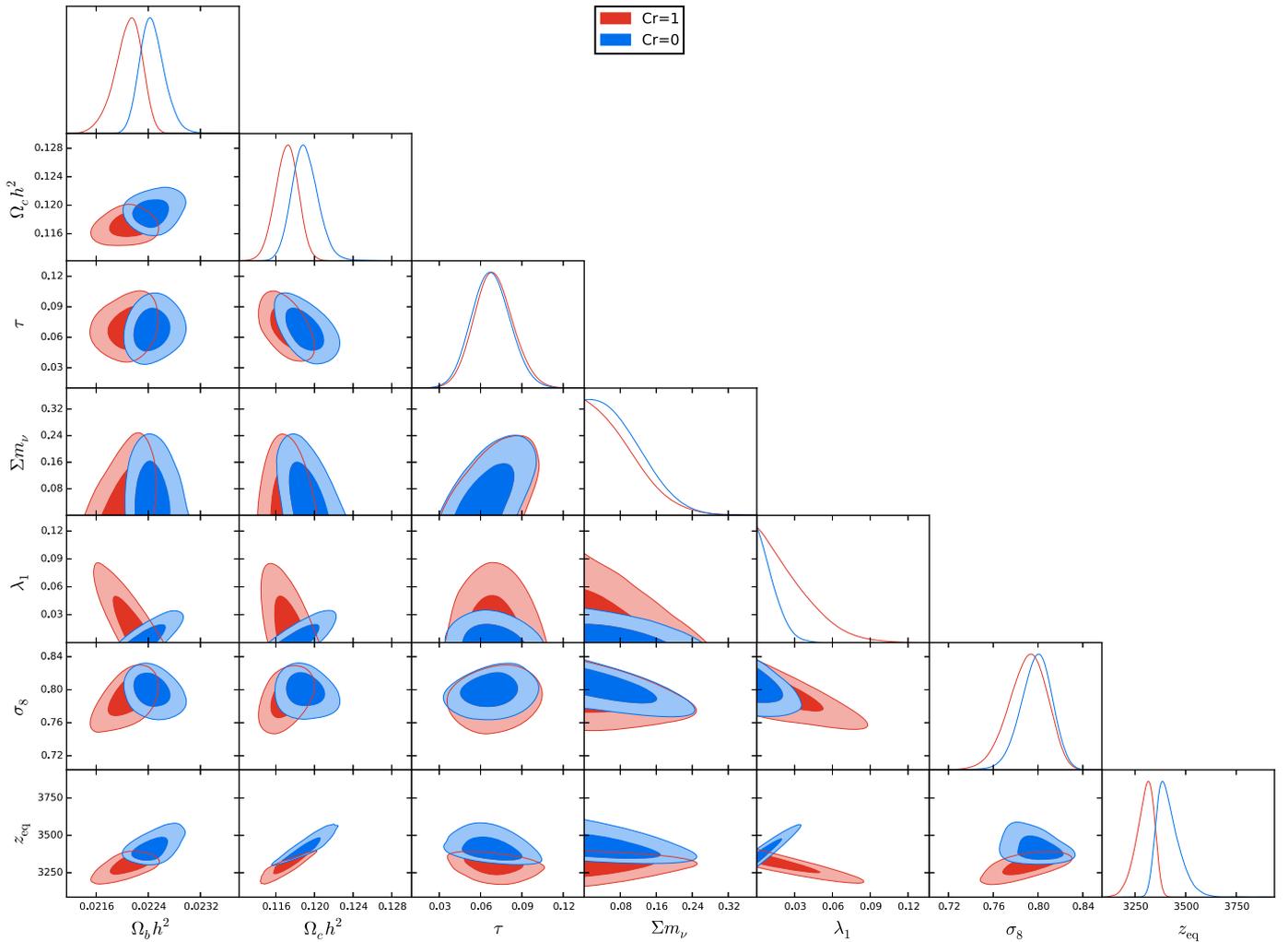


Fig. 5. One and two-dimensional distributions of $\Omega_b h^2$, $\Omega_c h^2$, τ , Σm_ν , λ_1 , σ_8 and z_{eq} , where the contour lines represent 68% and 95% confidence levels, respectively. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Table 2

List of allowed regions as 95% C.L. with $\rho_\Lambda = \rho_\Lambda^0 [\lambda_0 + \lambda_1 H/H_0]$ and $\lambda_0 + \lambda_1 = 1$, where $\chi_{\text{Best-fit}}^2 = \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{MPK}}^2 + \chi_{\text{lensing}}^2$.

Parameter	$\Lambda(t)$ CDM with $C_r = 1$	$\Lambda(t)$ CDM with $C_r = 0$	Λ CDM
Model parameter (λ_1)	$< 6.68 \times 10^{-2}$	$< 2.63 \times 10^{-2}$	0
Baryon density ($100\Omega_b h^2$)	$2.21_{-0.05}^{+0.04}$	2.25 ± 0.04	2.23 ± 0.04
CDM density ($\Omega_c h^2$)	0.117 ± 0.002	$0.119_{-0.002}^{+0.003}$	0.118 ± 0.002
Optical depth (τ)	$6.71_{-2.62}^{+2.83} \times 10^{-2}$	$6.74_{-2.62}^{+2.79} \times 10^{-2}$	$6.99_{-2.77}^{+2.83} \times 10^{-2}$
Neutrino mass sum (Σm_ν)	< 0.189 eV	< 0.194 eV	< 0.221 eV
σ_8	$0.791_{-0.035}^{+0.034}$	$0.799_{-0.029}^{+0.025}$	$= 0.805_{-0.028}^{+0.026}$
z_{eq}	3298_{-94}^{+82}	3414_{-98}^{+117}	3351 ± 46
$\chi_{\text{Best-fit}}^2$	13546.5	13545.7	13545.3
χ_{CMB}^2	13032.4	13032.2	13031.1
χ_{BAO}^2	4.3	4.3	4.8
χ_{MPK}^2	479.9	479.6	480.0
χ_{lensing}^2	29.9	29.5	29.4

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