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Folding of chaotic fractal space time

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Abstract

In this article, we introduce types of foldings in chaotic special types of space time. The effect of the foldings on the pure chaotic special space time is obtained. The limits of the foldings of special space time are achieved. The folding restricted on the charge and the mass are presented. The variations of the dimension of non-Riemannian chaotic manifold under the limit of folding are deduced. Types of the fractal foldings of the chaotic special space time are presented. Some applications concerning these relations are presented.

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1. Introduction and definitions

As is well known, the theory of folding is always one of interesting topics in Euclidian and Non-Euclidian space and it has been investigated from the various viewpoints by many branches of topology and differential geometry [5–24].

Deterministic chaos is now part of most scientific research, in varied fields. A big theory of everything has been discovered. Mathematicianes, physicists, and even biologists now have a common ground to work together. Chaos has revolutionized the way scientists think of nature, and the world around us [3,4,31]. Devaney describes a system as chaotic if there is (i) sensitivity to initial conditions, (ii) topological transitivity, and (iii) density of periodic points. He says that mathematical definitions approximate the idea of chaos, but do not capture it. The modern study of chaos began with the realization in 1960s that quit simple mathematical equations could model systems that were very complex. The simplest systems can produce extraordinarily difficult problems of predictability. Order may arise spontaneously in a system and from chaos spontaneous order which is called selforganized criticality [4,11,13,16]. Fractal is the geometry and patterns of deterministic chaos. For many the word fractal brings to mind the aesthetically pleasing, colorful images that are generated through computer graphics. Mandelbrott used the word to refer to fractional dimension [32]. The

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dimension of Euclidean geometry are 0, 1, 2, and 3 which are integers from the set $\{\ldots, -3, -2, -1, 1, 2, 3, \ldots\}$. The point in Euclidean geometry represents no or zero dimensions, the line represnts one dimension, the plane represents two dimensions, and space represents three dimensions. Fractal dimension is a fractional or partial dimension such as 1.45, which lies between the integers 1 and 2 [32].

We adapted these definitions and concepts to fit coherently into the framework of this article.

Definition 1. For Riemannian manifolds M and N (not necessarily of the same dimension) a map $\phi{:}M \to N$ is said to be a topological folding of M into N if, for each piecewise geodesic path $\gamma{:}I \to M$ ($I=[0,1] \subseteq R$), the induced path $\phi o \gamma{:}I \to N$ is a piecewise geodesic. If, in addition, $\phi{:}M \to N$ preserves lengths of paths, we call $\phi{:}M \to N$ an isometric folding of M into N. If the continuous map $\phi{:}M \to N$ is a folding then $dimM \le dimN$. Many types of foldings are discussed in [1,5,9,14,17,19,23]. Some applications are disscussed in [6,8,16,30].

Definition 2. If X is a topological space and $A \subset X$, there exist the continuous map r such that $r{:}X \to A$, $r(a)=a \forall a \in A$, So r is retraction function [2,22,25–27]. Many types of retractions are presented in [7,10,12,18].

Definition 3. A chaotic special space time B_i is the special space time B_i carries many physical characters. Each character represents special space time B_i homeomorphic to the orginal one [7,14,15].

2. Space time

The energy distribution of a static spherically symmetric charged black hole which the line element representing this special space time by [14,19,28,29].

$$dS^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - \left(1 - \frac{\alpha}{r}\right) r^2 (d\theta^2 + \sin^2\theta d\Phi^2) \tag{1}$$

where $\alpha = Q^2(\exp(-2\Phi_o)/M)$, M and Q are respectively mass and charged parameters; Φ_o is the asymptotic value of dilation field.

3. Main results

In what follows we would like to introduce types of conditional foldings of chaotic special space time which the metric is defined as:

$$dS_i^2 = \left(1 - \frac{2M_i}{r(\eta)}\right) dt^2(\eta) - \left(1 - \frac{2M_i}{r(\eta)}\right)^{-1} dr^2(\eta) - \left(1 - \frac{\alpha_i}{r(\eta)}\right) r^2(\eta)(d\theta^2(\eta) + \sin^2\theta(\eta) d\varphi^2(\eta)) \tag{2}$$

where $t(\eta)$, $r(\eta)$,$\theta(\eta)$ and $\phi(\eta)$ are functions of energy distribution, also $i=0, 1, 2, \ldots \infty$, if $i=0$ it is the space time. If $i=1, 2, \ldots \infty$ then dS_i^2 represents the special pure chaotic ones. Also, $\alpha_o = Q_0^2(\exp(-2\phi_o)/M_o)$, $\alpha_1 = Q_1^2(\exp(-2\phi_o))/M_1, \ldots \alpha_\infty = Q_\infty^2(\exp(-2\phi_o))/M_\infty$. Moreover, the coordinate of the chaotic special space time are:

$$\chi_{i1} = \sqrt{\left(1 - \frac{2M_i}{r(\eta)}\right) t^2(\eta) + C_1}$$

$$\chi_{i2} = \sqrt{C_2 - (r^2(\eta) + 4M_i r(\eta) + 3M_i^2 \ln(r(\eta) - 2M_i))}$$

$$\chi_{i3} = \sqrt{C_3 - \left(1 - \frac{\alpha_i}{r(\eta)}\right) r^2(\eta)\theta^2(\eta)}$$

$$\chi_{i4} = \sqrt{C_4 - \left(1 - \frac{\alpha_i}{r(\eta)}\right) r^2(\eta) \sin^2\theta(\eta)\phi^2(\eta)}$$

where C_1, C_2, C_3, and C_4 are the constant of integration.

Now we are going to introduce the conditional folding F_{im} of the chaotic special space time B_i into itself, let $F_{im}:B_i \to B_i$ such that

$$F_{om}(\chi_{01}, \chi_{02}, \chi_{03}, \chi_{04}) = \left(\frac{(|x_{o1}|)}{m}, \chi_{02}, \chi_{03}, \chi_{04}\right), \quad m = 1, 2, \ldots$$

An isometric chain folding of F_{0m} into itself may be defined by:

$$F_{01}: \quad \left\{\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))},\right.$$
$$\left.\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right\}$$
$$\to \left\{\left|\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))},\right.$$
$$\left.\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right\}$$

$$F_{02}: \left\{\left|\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}\right.$$
$$\left.\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right\}$$
$$\to \left\{\left|\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))},\right.$$
$$\left.\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)},\right\}, \ldots,$$

$$F_{0m}: \left\{\left|\frac{\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}}{m-1}\right|,\right.$$
$$\sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}, \sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)},$$
$$\left.\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right\}$$
$$\to \left\{\left|\frac{\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}}{m}\right|, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))},\right.$$
$$\left.\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right\}$$

induce an infinite number of pure chaotic folding of chaotic special space time defined as:

$$\bar{F}_{1m}(x_{11}, x_{12}, x_{13}, x_{14}) = \left(\frac{|x_{11}|}{m}, x_{12}, x_{13}, x_{14}\right),$$

$$\bar{F}_{2m}(x_{21}, x_{22}, x_{23}, x_{24}) = \left(\frac{|x_{21}|}{m}, x_{22}, x_{23}, x_{24}\right), \ldots,$$

$$\bar{F}_{\infty m}(x_{\infty 1}, x_{\infty 2}, x_{\infty 3}, x_{\infty 4}) = \left(\frac{|x_{\infty 1}|}{m}, x_{\infty 2}, x_{\infty 3}, x_{\infty 4}\right),$$

From the above discussion we have the following theorem.

Theorem 1. *Any conditional folding of the geometric special space time into itself induces an ininite number of foldings of the conditionals of the pure chaotic ones into themselves.*

Corollary 1. *The converse of the above theorem is not true.*

Theorem 2. *The limit of foldings of the chaotic special space time is a chaotic hypersurface special space time* $B_{i1} \subset B_i$

Proof. From Theorem (1) the limit folding of special space time is given by:

$$\lim_{m\to\infty} 0m \left(\left| \sqrt{\frac{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}{m}} \right|, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln((\eta)-2M_0))}, \right.$$

$$\left. \sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)} \right)$$

$$= \left\{0, \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}, \sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}, \right.$$

$$\left. \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)} \right\}$$

induce an infinite number of limits foldings of pure chaotic special space time defined as:

$$\lim_{m\to\infty} 1m \left(\frac{|x_{11}|}{m}, x_{12}, x_{13}, x_{14}\right) = (0, x_{12}, x_{13}, x_{14})$$

$$\lim_{m\to\infty} 2m \left(\frac{|x_{21}|}{m}, x_{22}, x_{23}, x_{24}\right) = (0, x_{22}, x_{23}, x_{24}), \ldots,$$

$$\lim_{m\to\infty} \infty m \left(\frac{|x_{\infty 1}|}{m}, x_{\infty 2}, x_{\infty 3}, x_{\infty 4}\right) = (0, x_{\infty 2}, x_{\infty 3}, x_{\infty 4})$$

which is the chaotic hypersurface special space time $B_{i1} \subset B_i$ □

Corollary 2. *The geometric limits foldings of special space time induce chaotic limits foldings of chaotic special space time.*

Proof. $\lim_{m\to\infty} F_{0m} \Rightarrow \lim_{m\to\infty} \bar{F}_{1m}, \lim_{m\to\infty} \bar{F}_{2m}, \ldots, \lim_{m\to\infty} \bar{F}_{\infty m}$.

Now, let $f_{on} : B_i \to B_i$ be given by $f_{on}(x_{01}, x_{02}, x_{03}, x_{04}) = \left(\frac{|x_{01}|}{n}, \frac{|x_{02}|}{n}, \frac{|x_{03}|}{n}, \frac{|x_{04}|}{n}\right), n = 1, 2, \ldots$

An isometric folding of B_i into itself may be defined by:

$$\mathrm{f}_{o1}:\quad \left\{\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1},\ \sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0)),}\right.$$

$$\left.\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)},\ \sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right\}$$

$$\left\{\left|\sqrt{\left(1-\frac{2M_i}{r(\eta)}\right)t^2(\eta)+C_1}\right|,\ \left|\sqrt{C_2-r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0)}\right|,\right.$$

$$\left.\left|\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|,\ \left|\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|\right\}$$

$$\mathrm{f}_{o2}:\left\{\left|\sqrt{\left(1-\frac{2M_i}{r(\eta)}\right)t^2(\eta)+C_1}\right|,\ \left|\sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}\right|,\right.$$

$$\left.\left|\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|,\ \left|\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|\right\}$$

$$\to\left\{\frac{\left|\sqrt{(1-\left(\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{2},\ \frac{\left|\sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}\right|}{2},\right.$$

$$\left.\frac{\left|\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|}{2},\ \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{2}\right\},\ldots,$$

$$\mathrm{f}_{on}:\quad\left\{\frac{\left|\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{n-1},\ \frac{\left|\sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}\right|}{n-1},\right.$$

$$\left.\frac{\left|\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|}{n-1},\ \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{n-1}\right\}$$

$$\to\left\{\frac{\left|\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{n},\ \frac{\left|\sqrt{C_2-(r^2(\eta))+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0)}\right|}{n},\right.$$

$$\left.\frac{\left|\sqrt{C_3-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|}{n},\ \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{n}\right\}$$

Then we have

$$\lim_{n\to\infty} f_{0n}\left\{\frac{\left|\sqrt{\left(1-\frac{2M_0}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{n}, \frac{\left|\sqrt{C_2-(r^2(\eta)+4M_0r(\eta)+3M_0^2\ln(r(\eta)-2M_0))}\right|}{n},\right.$$

$$\left.\frac{\left|\sqrt{C_3-(1-(\alpha_0/r(\eta)))r^2(\eta)\theta^2(\eta)}\right|}{n}, \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_0}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{n}\right\} = \{O_{01}, O_{02}, O_{03}, O_{04}\}$$

Also, we have

$$\lim_{n\to\infty} f_{1n}\left\{\frac{\left|\sqrt{\left(1-\frac{2M_1}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{n}, \frac{\left|\sqrt{C_2-(r^2(\eta)+4M_1r(\eta)+3M_1^2\ln(r(\eta)-2M_1))}\right|}{n},\right.$$

$$\left.\frac{\left|\sqrt{C_3-\left(1-\frac{\alpha_1}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|}{n}, \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_1}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{n}\right\} = \{O_{11}, O_{12}, O_{13}, O_{14}\}$$

$$\lim_{n\to\infty} f_{2n}\left\{\frac{\left|\sqrt{\left(1-\frac{2M_2}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{n}, \frac{\left|\sqrt{C_2-(r^2(\eta)+4M_2r(\eta)+3M_2^2\ln(r(\eta)-2M_2))}\right|}{n},\right.$$

$$\left.\frac{\left|\sqrt{C_3-\left(1-\frac{\alpha_2}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|}{n}, \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_2}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{n}\right\} = \{O_{21}, O_{22}, O_{23}, O_{24}\}, \ldots,$$

$$\lim_{n\to\infty} f_{\infty n}\left\{\frac{\left|\sqrt{\left(1-\frac{2M_\infty}{r(\eta)}\right)t^2(\eta)+C_1}\right|}{n}, \frac{\left|\sqrt{C_2-(r^2(\eta)+4M_\infty r(\eta)+3M_\infty^2\ln(r(\eta)-2M_\infty))}\right|}{n},\right.$$

$$\left.\frac{\left|\sqrt{C_3-\left(1-\frac{\alpha_\infty}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)}\right|}{n}, \frac{\left|\sqrt{C_4-\left(1-\frac{\alpha_\infty}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)}\right|}{n}\right\}$$

$$= \{O_{\infty 1}, O_{\infty 2}, O_{\infty 3}, O_{\infty 4}\}, \ldots,$$

□

Thus, the following theorem is obtained:

Theorem 3. *The end of limits of foldings of special space time into itself induces an infinite number of end of limits of the similar foldings of pure chaotic ones.*

Corollary 3. *The above end of limits of foldings of special space time equivalent to the minimal retraction of the same special space time.*

Now, let $\gamma_{01}: \quad B_{0i}^{n} \rightarrow B_{0i}^{n}$ be a topological folding of n-dimensional chaotic special space time into itself,

$$\gamma_{02}: \gamma_{01}(B_{0i}^{n}) \rightarrow \gamma_{01}(B_{0i}^{n}), \quad \gamma_{03}: \gamma_{02}(\gamma_{01}(B_{0i}^{n})) \rightarrow \gamma_{02}(\gamma_{01}(B_{0i}^{n})), \ldots,$$

$$\gamma_{0n}: \gamma_{0n-1}(\gamma_{0n-2}\ldots\gamma_{02}(\gamma_{01}(B_{0i}^{n}))\cdots) \rightarrow \gamma_{0n-1}(\gamma_{0n-2}\ldots\gamma_{02}(\gamma_{01}(B_{0i}^{n}))\cdots)$$

$$\lim_{x\to\infty}\gamma_{0n}(\gamma_{0n-1}(\gamma_{0n-2}\cdots\gamma_{02}(\gamma_{01}(B_{0i}^{n}))\cdots)) = B_{oi}^{n-1}$$

which is the chaotic special space time of dimension $(n-1)$

Also, let

$$\bar{\gamma}_{01}: B_{0i}^{n-1} \rightarrow B_{0i}^{n-1}, \quad \bar{\gamma}_{02}: \bar{\gamma}_{01}(B_{0i}^{n-1}) \rightarrow \bar{\gamma}_{01}(B_{0i}^{n-1})$$

$$\bar{\gamma}_{03}: \bar{\gamma}_{02}(\bar{\gamma}_{01}(B_{0i}^{n-1})) \rightarrow \bar{\gamma}_{02}(\bar{\gamma}_{01}(B_{0i}^{n-1}))$$

$$\bar{\gamma}_{0m}: \bar{\gamma}_{0(m-1)}(\bar{\gamma}_{0(m-2)}\cdots\bar{\gamma}_{02}(\bar{\gamma}_{01}(B_{0i}^{n-1}))\cdots) \rightarrow \bar{\gamma}_{0(m-1)}(\bar{\gamma}_{0(m-2)}\cdots\bar{\gamma}_{02}(\bar{\gamma}_{01}(B_{0i}^{n-1}))\cdots),$$

$$\lim_{m\to\infty}\bar{\gamma}_{0m}(\bar{\gamma}_{0(m-1)}(\bar{\gamma}_{0(m-2)})\cdots\bar{\gamma}_{02}(\bar{\gamma}_{01}(B_{0i}^{n-1}))\cdots) = B_{0i}^{n-2}$$

which is the chaotic special space time of dimension $(n-2)$.

Consequently $\lim_{s\to\infty}\lim_{m\to\infty}\lim_{n\to\infty}\cdots\bar{\bar{\gamma}}_{0s}(\bar{\gamma}_{0m}(\gamma_{0n}\cdots\gamma_{02}(\gamma_{01}(B_{0i}^{n}))\cdots)) = B_{0i}^{O}$

Moreover, the end of limits of foldings of special space time induce an infinite number of end of limits of foldings of chaotic special space time, i.e.

$$\lim_{m\to\infty}\gamma_{11}(\gamma_{12}(\gamma_{13}\cdots\gamma_{1\infty}(B_{0i}^{0})\cdots)) = B_{1i}^{0}$$

$$\lim_{m\to\infty}\gamma_{21}(\gamma_{22}(\gamma_{23}\cdots\gamma_{2\infty}(B_{0i}^{0})\cdots)) = B_{2i}^{0}$$

$$\lim_{m\to\infty}\gamma_{31}(\gamma_{32}(\gamma_{33}\cdots\gamma_{3\infty}(B_{0i}^{0})\cdots)) = B_{3i}^{0}, \ldots,$$

$$\lim_{m\to\infty}\gamma_{\infty 1}(\gamma_{\infty 2}(\gamma_{\infty 3}\cdots\gamma_{\infty\infty}(B_{0i}^{0})\cdots)) = B_{\infty i}^{0}$$

Now, we are in a position to formulate the following theorem:

Theorem 4. *The end of the limits of the foldings of chaotic special space time of dimension n is a o-dimensioal chaotic special space time induces an infinite number of end of limits of foldings of chaotic ones.*

Now, let $\upsilon_{01}: Q_{12\ldots\infty h}^{n} \rightarrow Q_{12\ldots\infty h}^{n}$ be a topological folding of charge of black hole into itself,

$$\upsilon_{02}: \upsilon_{01}(Q_{12\ldots\infty h}^{n}) \rightarrow \upsilon_{01}(Q_{12\ldots\infty h}^{n}),\ \upsilon_{03}: \upsilon_{02}(\upsilon_{01}(Q_{12\ldots\infty h}^{n})) \rightarrow \upsilon_{02}(\upsilon_{01}(Q_{12\ldots\infty h}^{n})), \ldots, \upsilon_{0n}: \upsilon_{0(n-1)}$$

$$(\upsilon_{0(n-2)}\ldots\upsilon_{02}(\upsilon_{01}(Q_{12\ldots\infty h}^{n}))\ldots) \rightarrow \upsilon_{0(n-1)}(\upsilon_{0(n-2)}\ldots\upsilon_{02}(\upsilon_{01}(Q_{12\ldots\infty h}^{n}))\ldots)$$

$$\lim_{n\to\infty}\upsilon_{0n}(\upsilon_{0(n-1)}\ldots\upsilon_{02}(\upsilon_{01}(Q_{12\ldots\infty h}^{n}))\ldots) = Q_{23\ldots\infty h}^{n-1},$$

Also, let $\bar{\upsilon}_{01}: Q_{23\ldots\infty h}^{n-1} \rightarrow Q_{23\ldots\infty h}^{n-1}, \bar{\upsilon}_{02}: \bar{\upsilon}_{01}(Q_{23\ldots\infty h}^{n-1}) \rightarrow \bar{\upsilon}_{01}(Q_{23\ldots\infty h}^{n-1}), \ldots,$

$$\bar{\upsilon}_{0m}: \bar{\upsilon}_{0(m-1)}(\bar{\upsilon}_{0(m-2)}\ldots\bar{\upsilon}_{02}(\bar{\upsilon}_{01}(Q_{23\ldots\infty h}^{n-1}))\ldots) \rightarrow \bar{\upsilon}_{0(m-1)}(\bar{\upsilon}_{0(m-2)}\ldots\bar{\upsilon}_{02}(\bar{\upsilon}_{01}(Q_{23\ldots\infty h}^{n-1}))\ldots),$$

$$\lim_{m\to\infty}\bar{\upsilon}_{0(m-1)}\ldots(\bar{\upsilon}_{02}(\bar{\upsilon}_{01}(Q_{23\ldots\infty h}^{n-1}))\ldots)) = Q_{34\ldots\infty h}^{n-2}.$$

Consequently,

$$\lim_{s\to\infty}\lim_{m\to\infty}\lim_{n\to\infty}\bar{\bar{\upsilon}}_{0s}(\bar{\upsilon}_{0m}(\upsilon_{0n}\ldots\upsilon_{02}(\upsilon_{01}(Q_{12\ldots\infty h}^{n}))\ldots) = Q_{0}^{0}$$

Thus, the following theorem is obtained:

Theorem 5. *The end of the limits of foldings of charge on some chaotic black hole is zero dimensional charge.*

Now, let $\phi : m^n_{12\ldots\infty h} \to m^n_{12\ldots\infty h}$ be a type of folding of n-dimensional mass $m^n_{12\ldots\infty h}$ black hole. then, we have the following chain

$$m^n_{12\ldots\infty h} \xrightarrow{\varphi_1^1} m^n_{12\ldots\infty h} \xrightarrow{\varphi_2^1} m^n_{12\ldots\infty h} \cdots m^n_{12\ldots\infty h} \overset{\lim\limits_{n\to\infty} \varphi_1^1}{\longrightarrow} \varphi_n^1 m^n_{23\ldots\infty h}$$

$$m^n_{23\ldots\infty h} \xrightarrow{\varphi_1^2} m^n_{23\ldots\infty h} \xrightarrow{\varphi_2^2} m^n_{23\ldots\infty h} \cdots \overset{\lim\limits_{n\to\infty} \varphi_1^2}{\longrightarrow} m^n_{34\ldots\infty h}$$

$$m^n_{34\ldots\infty h} \xrightarrow{\varphi_1^3} m^n_{34\ldots\infty h} \xrightarrow{\varphi_2^3} m^n_{34\ldots\infty h} \cdots m^n_{34\ldots\infty h} \overset{\lim\limits_{n\to\infty} \varphi_n^3}{\longrightarrow} \varphi_n^1 m^n_{45\ldots\infty h}, \ldots,$$

$$m^n_{0h} \xrightarrow{\varphi_1^n} m^n_{0h} \xrightarrow{\varphi_2^n} m^n_{0h} \cdots m^n_{0h} \overset{\lim\limits_{n\to\infty} \varphi_n^n}{\longrightarrow} m^n_0$$

Now, we are in a position to formulate the following theorem.

Theorem 6. *The end of the limits of foldings of n-dimentional mass chaotic black hole is a type of the same dimension space time.*

Fractal folding of the chaotic special space time.

Our aim, in the present section is to introduce two types of fractal folding:

(a) Fractal folding from $B_{0h}^{4+(p/q)}$-dimensional chaotic special space time to B_{0h}^4-dimensional chaotic special space time.

Now, we will define the type of folding which reduces the dimension from fractal to integer,
let

$$f : B_{0h}^{4+(p/q)} \to B_{0h}^{4+(p/q)},\ f_{01}(x_{01}+\zeta_1, x_{02}+\zeta_2, x_{03}+\zeta_3, x_{04}+\zeta_4) = (x_{01}+\zeta_1^1, x_{02}+\zeta_2^1, x_{03}+\zeta_3^1, x_{04}+\zeta_4^1),$$

$$f_{02}(x_{01}+\zeta_1^1, x_{02}+\zeta_2^1, x_{03}+\zeta_3^1, x_{04}+\zeta_4^1) = (x_{01}+\zeta_1^2, x_{02}+\zeta_2^2, x_{03}+\zeta_3^2, x_{04}+\zeta_4^2)$$

$$,\ldots,$$

$$f_{0n}(x_{01}+\zeta_1^{n-1}, x_{02}+\zeta_2^{n-1}, x_{03}+\zeta_3^{n-1}, x_{04}+\zeta_4^{n-1}) = (x_{01}+\zeta_1^n, x_{02}+\zeta_2^n, x_{03}+\zeta_3^n, x_{04}+\zeta_4^n)$$

where

$$\zeta_1^n < \zeta_1^{n-1} < \zeta_1^{n-2} < \cdots < \zeta_1^2 < \zeta_1^1,$$
$$\zeta_2^n < \zeta_2^{n-1} < \zeta_2^{n-2} < \cdots < \zeta_2^2 < \zeta_2^1,$$
$$\zeta_3^n < \zeta_3^{n-1} < \zeta_3^{n-2} < \cdots < \zeta_3^2 < \zeta_3^1,$$
$$\zeta_4^n < \zeta_4^{n-1} < \zeta_4^{n-2} < \cdots < \zeta_4^2 < \zeta_4^1,$$

$$\lim_{n\to\infty} f \quad on(B_{0h}^{4+(p/q)}) = B_{0h}^4$$

There is a corresponding induced sequence of fractal folding $\bar{f}_1 : B_{1h}^{-4+(p/q)} \to B_{1h}^{-4+(p/q)}$

$$\bar{f}_{11}(x_{11}+\zeta_{11}, x_{12}+\zeta_{12}, x_{13}+\zeta_{13}, x_{14}+\zeta_{14}) = (x_{11}+\zeta_{11}^1, x_{12}+\zeta_{12}^1, x_{13}+\zeta_{13}^1, x_{14}+\zeta_{14}^1),$$

$$\bar{f}_{12}(x_{11}+\zeta_{11}^1, x_{12}+\zeta_{12}^1, x_{13}+\zeta_{13}^1, x_{14}+\zeta_{14}^1) = (x_{11}+\zeta_{11}^2, x_{12}+\zeta_{12}^2, x_{13}+\zeta_{13}^2, x_{14}+\zeta_{14}^2), \ldots,$$

$$\bar{f}_{1n}(x_{11}+\zeta_{11}^{n-1}, x_{12}+\zeta_{12}^{n-1}, x_{13}+\zeta_{13}^{n-1}, x_{14}+\zeta_{14}^{n-1}) = (x_{11}+\zeta_{11}^n, x_{12}+\zeta_{12}^n, x_{13}+\zeta_{13}^n, x_{14}+\zeta_{14}^n),$$

where

$$\zeta_{11}^{n} < \zeta_{11}^{n-1} < \zeta_{11}^{n-2} < \cdots < \zeta_{11}^{2} < \zeta_{11}^{1},$$
$$\zeta_{12}^{n} < \zeta_{12}^{n-1} < \zeta_{12}^{n-2} < \cdots < \zeta_{12}^{2} < \zeta_{12}^{1},$$
$$\zeta_{13}^{n} < \zeta_{13}^{n-1} < \zeta_{13}^{n-2} < \cdots < \zeta_{13}^{2} < \zeta_{13}^{1},$$
$$\zeta_{14}^{n} < \zeta_{14}^{n-1} < \zeta_{14}^{n-2} < \cdots < \zeta_{14}^{2} < \zeta_{14}^{1},$$

$$\lim_{n\to\infty} \bar{f}_{1n}(B_{ih}^{4+(p/q)}) = B_{ih}^{4}$$

Now, we are in a position to formulate the following theorem:

Theorem 7. *The geometric fractal folding of special space time into itself which reduces the dimension from fractal to integer induces an infinite number of pure chaotic fractal folding of chaotic special space time into itself but the converse is not true.*

(b) Fractal folding from B_{0h}^{4}-dimensional special space time to B_{0h}^{3}-dimensional special space time. It is the folding which reduces the dimension from integer to fractal.

Now, let $f_0 : B_{0h}^{4} \to B_{0h}^{4}$

$$f_{01}(\chi_{01}, \chi_{02}, \chi_{03}, \chi_{04}) = (\chi_{01}, \chi_{02}, \chi_{03}, \zeta_1),$$

$$f_{02}(\chi_{01}, \chi_{02}, \chi_{03}, \zeta_1) = (\chi_{01}, \chi_{02}, \chi_{03}, \zeta_2), \ldots,$$

$$f_{0n}(\chi_{01}, \chi_{02}, \chi_{03}, \zeta_{n-1}) = (\chi_{01}, \chi_{02}, \chi_{03}, \zeta_n).$$

where

$$\zeta_n < \zeta n - 1 < \cdots < \zeta_2 < \zeta_1, \lim_{n\to\infty} fon(x_{01}, x_{02}, x_{03}, \zeta_n) = (x_{01}, x_{02}, x_{03}, 0).$$

This limit will be defined when $C_4 \to 0$ and $\phi \to 0$, where the coordinate of the special space time are:

$$x_{01} = \sqrt{\left(1 - \frac{2M}{r(\eta)}\right) t^2(\eta) + C_1}$$

$$x_{02} = \sqrt{C_1 - (r^2(\eta) + 4Mr(\eta) + 3M^2 \mathrm{In}(r(\eta)) - 2M))}$$

$$x_{03} = \sqrt{C_3 - \left(1 - \frac{\alpha}{r(\eta)}\right) r^2(\eta)\theta^2(\eta)}$$

$$x_{04} = \sqrt{C_4 - \left(1 - \frac{\alpha}{r(\eta)}\right) r^2(\eta) \sin^2\theta(\eta)\phi^2(\eta)}$$

Also, let $g_0 : B_{0h}^{3} \to B_{0h}^{3}$

$$g_{01}(x_{01}, x_{02}, x_{03}, 0) = (x_{01}, x_{02}, \zeta_1, 0),$$

$$g_{02}(x_{01}, x_{02}, \zeta_1, 0) = (x_{01}, x_{02}, \zeta_2, 0), \ldots,$$

$$g_{0n}(x_{01}, x_{02}, \zeta_{n-1}, 0) = (x_{01}, x_{02}, \zeta_n, 0).$$

where $\zeta_n < \zeta_{n-1} < \cdots < \zeta_2 < \zeta_1$, $\lim_{n\to\infty} gon(x_{01}, x_{02}, \zeta_n, 0) = (x_{01}, x_{02}, 0, 0)$. This limit defined when $C_3 \to 0$ and $\theta \to 0$.

Now, let $k_0 : B^2_{0h} \to B^2_{0h}$,

$$k_{01}(x_{01}, x_{02}, 0, 0) = (x_{01}, \zeta_1, 0, 0),$$

$$k_{02}(x_{01}, \zeta_1, 0, 0) = (x_{01}, \zeta_2, 0, 0), ...,$$

$$k_{0n}(x_{01}, \zeta_{n-1}, 0, 0) = (x_{01}, \zeta_n, 0, 0),$$

$$\zeta_n < \zeta_{n-1} < \cdots < \zeta_2 < \zeta_1,$$

$$\lim_{n\to\infty} kon(x_{01}, \zeta_n, 0, 0) = (x_{01}, 0, 0, 0).$$

Moreover, let $R_0 : B^1_{0h} \to B^1_{0h}$

$$R_{01}(x_{01}, 0, 0, 0) = (\zeta_1, 0, 0, 0),$$

$$R_{02}(\zeta_1, 0, 0, 0) = (\zeta_2, 0, 0, 0), \ldots,$$

$$R_{0n}(\zeta_{n-1}, 0, 0, 0) = (\zeta_n, 0, 0, 0),$$

$$\lim_{n\to\infty} Ron(\zeta_n, 0, 0, 0) = (0, 0, 0, 0)$$

Also, this limit is defined when $C_1 \to 0$ and $t \to 0$.

There is a corresponding induced sequence of fractal folding:

$$\bar{f}_1 : \bar{B}^4_{1h} \to \bar{B}^4_{1h},$$

$$\bar{f}_{11}(x_{11}, x_{12}, x_{13}, x_{14}) = (x_{11}, x_{12}, x_{13}, \bar{\zeta}^1_1),$$

$$\bar{f}_{12}(x_{11}, x_{12}, x_{13}, \bar{\zeta}^1_1) = (x_{11}, x_{12}, x_{13}, \bar{\zeta}^1_2), \ldots,$$

$$\bar{f}_{1n}(x_{11}, x_{12}, x_{13}, \bar{\zeta}^1_{n-1}) = (x_{11}, x_{12}, x_{13}, \bar{\zeta}^1_n), \bar{\zeta}^1_n < \bar{\zeta}^1_{n-1} < \cdots < \bar{\zeta}^1_2 < \bar{\zeta}^1_1$$

$$\lim_{n\to\infty} \bar{f}_{1n}(x_{11}, x_{12}, x_{13}, \bar{\zeta}^1_n) = (x_{11}, x_{12}, x_{13}, 0),$$

Also, let $\bar{f}_2 : \bar{B}^4_{2h} \to \bar{B}^4_{2h}$.

$$\bar{f}_{21}(x_{21}, x_{22}, x_{23}, x_{24}) = (x_{21}, x_{22}, x_{23}, \bar{\zeta}^2_1),$$

$$\bar{f}_{22}(x_{21}, x_{22}, x_{23}, \bar{\zeta}^2_1) = (x_{21}, x_{22}, x_{23}, \bar{\zeta}^2_2), \ldots,$$

$\bar{f}_{2n}(x_{21}, x_{22}, x_{23}, \bar{\zeta}^2_{n-1}) = (x_{21}, x_{22}, x_{23}, \bar{\zeta}^2_n)$, where $\bar{\zeta}^2_n < \bar{\zeta}^2_{n-1} < \cdots < \bar{\zeta}^2_2 < \bar{\zeta}^2_1$

$$\lim_{n\to\infty} \bar{f}_{2n}(x_{21}, x_{22}, x_{23}, \bar{\zeta}^2_n) = (x_{21}, x_{22}, x_{23}, 0), \ldots,$$

Also, let $\bar{f}_m : \bar{B}^4_{mh} \to \bar{B}^4_{mh}$.

$$\bar{f}_{m1}(x_{m1}, x_{m2}, x_{m3}, x_{m4}) = (x_{m1}, x_{m2}, x_{m3}, \bar{\zeta}^m_1),$$

$$\bar{f}_{m2}(x_{m1}, x_{m2}, x_{m3}, \bar{\zeta}^m_1) = (x_{m1}, x_{m2}, x_{m3}, \bar{\zeta}^m_2), ...,$$

$$\bar{f}_{mn}(x_{m1}, x_{m2}, x_{m3}, \bar{\zeta}_{n-1}^{m}) = (x_{m1}, x_{m2}, x_{m3}, \bar{\zeta}_{n}^{m}), \bar{\zeta}_{n}^{m} < \bar{\zeta}_{n-1}^{m} < \cdots < \bar{\zeta}_{2}^{m} < \bar{\zeta}_{1}^{m},$$

$$\lim_{n\to\infty} f_{mn}(x_{m1}, x_{m2}, x_{m3}, \bar{\zeta}_{n}^{m}) = (x_{m1}, x_{m2}, x_{m3}, 0),$$

Also, we have

$$\bar{g}_1 : \bar{B}_{1h}^3 \to \bar{B}_{1h}^3, \bar{g}_2 : \bar{B}_{2h}^3 \to \bar{B}_{2h}^3, ..., \bar{g}_m : \bar{B}_{mh}^3 \to \bar{B}_{mh}^3,$$

$$\bar{g}_{11}, \bar{g}_{12}, \ldots, \bar{g}_{1n}, \bar{g}_{21}, \bar{g}_{22}, \ldots, \bar{g}_{2n}, \ldots, \bar{g}_{m1}, \bar{g}_{m2}, \ldots, \bar{g}_{mn}$$

such that

$$\lim_{n\to\infty} \bar{g}_{1n}(\bar{B}_{1h}^3) = (\bar{B}_{1h}^2), \lim_{n\to\infty} \bar{g}_{2n}(\bar{B}_{2h}^3) = \bar{B}_{2h}^2, \ldots,$$

$$\lim_{n\to\infty} \bar{g}_{mn}(\bar{B}_{mh}^3) = (\bar{B}_{mh}^2).$$

Also, we have

$$\bar{k}_1 : \bar{B}_{1h}^2 \to \bar{B}_{1h}^2, \bar{k}_2 : \bar{B}_{2h}^2 \to \bar{B}_{2h}^2, \ldots, \bar{k}_m : \bar{B}_{mh}^2 \to \bar{B}_{mh}^2,$$

$$\bar{k}_{11}, \bar{k}_{12}, \ldots, \bar{k}_{1n}, \bar{k}_{21}, \bar{k}_{22}, \ldots, \bar{k}_{2n}, \ldots, \bar{k}_{m1}, \bar{k}_{m2}, \ldots, \bar{k}_{mn}$$

such that

$$\lim_{n\to\infty} \bar{k}_{mn}(\bar{B}_{1h}^2) = \bar{B}_{1h}^1, \lim_{n\to\infty} \bar{k}_{2n}(\bar{B}_{2h}^2) = \bar{B}_{2h}^1, \ldots, \lim_{n\to\infty} \bar{k}_{mn}(\bar{B}_{mh}^2) = \bar{B}_{mh}^1$$

Lastely, we have

$$\bar{R}_1 : \bar{B}_{1h}^1 \to \bar{B}_{1h}^1, \bar{R}_2 : \bar{B}_{2h}^1 \to \bar{B}_{2h}^1, \ldots, \bar{R}_m : \bar{B}_{mh}^1 \to \bar{B}_{mh}^1,$$

$$\bar{R}_{11}, \bar{R}_{12}, \ldots, \bar{R}_{1n}, \bar{R}_{21}, \bar{R}_{22}, \ldots, \bar{R}_{2n}, \ldots, \bar{R}_{m1}, \bar{R}_{m2}, \ldots, \bar{R}_{mn}$$

such that

$$\lim_{n\to\infty} \bar{R}_{1n}(\bar{B}_{1h}^1) = \bar{B}_{1h}^0, \lim_{n\to\infty} \bar{R}_{2n}(\bar{B}_{2h}^1) = \bar{B}_{2h}^0, \ldots, \lim_{n\to\infty} \bar{R}_{mn}(\bar{B}_{mh}^1) = \bar{B}_{mh}^0.$$

Hence, we can formulate the following theorems:

Theorem 8. *The fractal folding of the geometric chaotic special space time which reduces the dimension from integer to fractal induces a fractal dimensional of the pure chaotic special space time but the converse is not true.*

Theorem 9. *The end of the limit of fractal folding of type* (*b*) *equal to o-dimensional manifold.*

4. Applications

The value of the energy distribution depends on the mass m and the energy Q, for instance, Reissner-Nordstrom gives several definitions of energy give

$$E = m - \frac{Q^2}{2r} \quad [20], \tag{4.1}$$

$$E_{\mu ol} = m - \frac{Q^2}{2r} \quad [20] \tag{4.2}$$

It is proved that the limit of charge equal zero. Substituting that in Eqs. (4.1) and (4.2) we conclude that $E = m$. Also, Chamorro, Vibhadra and Xulu showed that using Einstein and Moller prescriptions, concerning the energy distribution of charge dilation black hole depends on the value λ:

$$E_{Einst} = m - \frac{Q^2}{2r}(1 - \lambda^2), \tag{4.3}$$

$$E_{\mu ol} = m - \frac{Q^2}{2r}(1 - \lambda^2), \tag{4.4}$$

where a dimensionless parameter λ controls the coupling between the dilation and Maxwell field [14,19,29]. It is proved that in the end of limit of charge equal zero. Substituting that in Eqs. (4.3) and (4.4) we conclude that $E = m$.

5. Conclusion

In this paper we have presented the variation of the folding on the charge Q, mass m and parameter λ in chaotic space time. The relation between the geodesic and folding are deduced. The folding of chaotic space time from view point of charge Q, mass m and dimensional parameter λ have been obtained. Also, the connection between the folding of the geometric and chaotic space time are deduced. Types of the fractal foldings of the chaotic space time are presented. The end of the limits of folding of charge Q, mass m and parameter λ are also discussed.

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