



On correcting the concentration index for binary variables

Gustav Kjellsson^{a,b,*}, Ulf-G. Gerdtham^{a,b,c}

^a Department of Economics, Lund University, P.O. Box 7082, SE-220 07 Lund, Sweden

^b Health Economics & Management, Institute of Economic Research, Lund University, P.O. Box 7080, SE-220 07 Lund, Sweden

^c Center for Primary Health Care Research, Malmö University Hospital, Lund University/Region Skåne, SE-205 02 Malmö, Sweden

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ABSTRACT

This article discusses measurement of socioeconomic inequalities in the prevalence of a health condition, in response to the recent exchange between Guido Erreygers and Adam Wagstaff, in which they discuss the merits of their own corrections to the frequently used concentration index. We first reconcile their debate and discuss the value judgments implicit in their indices. Next, we provide a formal definition of the previously undefined value judgment in Wagstaff's correction. Finally, we show empirically that the choice of index matters, as illustrated by comparisons between countries using data from the European Survey of Health, Ageing and Retirement.

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1. Introduction

Since the 1990s, the concentration index approach has been the standard tool in health economics for evaluating socioeconomic inequalities in health. Because the concentration index (C) is derived from the Gini coefficient of income inequalities, it requires the health variable to be on the same scale as income, i.e. on a ratio-scaled measure without an upper bound (Erreygers, 2009a). As health differs from income in several aspects, such a variable is rarely at hand. Instead, health measures tend to be bounded and either ordinal or cardinal.

For bounded variables, (1) C may rank countries by inequalities in health and ill-health differently (Clarke et al., 2002), (2) the maximum and minimum value of C depend on the mean health in the society (Wagstaff, 2005), and (3) the value of C depends on the scale of the health variable (Erreygers, 2009a). To account for these issues, Erreygers (2009a) and Wagstaff (2005) have each developed their own corrections of C for bounded variables.

The merits and value judgments of these indices have been intensely debated (Erreygers, 2009a,b; Erreygers and Van Ourti, 2011a,b; Wagstaff, 2005, 2009, 2011a,b). Erreygers (2009a,b)

claims that his index is superior because it satisfies level independence (i.e. an equal increment of health for all individuals does not affect the value of the index), while Wagstaff (2009) questions the desirability of that very property. Although we applaud Erreygers' quest to extend the concentration index to cardinal variables, we argue that Erreygers' index (E) is not necessarily superior to Wagstaff's (W) and the difference between the indices is normative rather than technical. Although the initial debate mainly concerned cardinal variables (Erreygers, 2009a,b; Wagstaff, 2009) and Erreygers and Van Ourti (2011a) take a more rigorous approach to discussing inequality measures for several types of variables, the latest exchange in *Health Economics* discusses binary variables (Erreygers and Van Ourti, 2011b; Wagstaff, 2011a,b). This article reconciles, and contributes to, this debate by further examining the methodological and empirical differences between Erreygers' and Wagstaff's suggested indices, focusing on binary variables.

Specifically, we formally express the previously undefined value judgment underlying W . The importance of further scrutinizing the implicit value judgments of the indices is evident as this exercise places the conclusions from the recent exchange in a completely different light. An in-depth discussion of these issues is important because many health variables of empirical interest (e.g. malnutrition, obesity, and a variety of self-assessed measures) are binary (see Hernández-Quevedo et al., 2006; Harper and Lynch, 2007; Van de Poel et al., 2008; Mackenbach et al., 2008), and researchers and practitioners may otherwise use an inequality measure

* Corresponding author at: Department of Economics, P.O. Box 7082, SE-220 07 Lund, Sweden. Tel.: +46 46 2227911; fax: +46 46 2224118.

E-mail address: gustav.kjellsson@nek.lu.se (G. Kjellsson).

without considering its implicit value judgments (e.g. Mischra and Joe, 2010; Hernández-Quevedo et al., 2010).

Following Erreygers (2009a,b) and Erreygers and Van Ourti (2011a), we acknowledge that the notion of relative and absolute value judgment changes when the health measure is no longer an unbounded ratio-scaled variable but a binary one. However, if we interpret the binary variable as a ratio-scaled projection of prevalence on an aggregated level, we may still use the language of relative and absolute.

As W , E , and C all condition absolute inequalities on different definitions of *the most unequal society*, they capture different perspectives on socioeconomic inequalities, and the choice of index therefore depends on the preferred value judgment. Because E weights absolute inequalities constantly and independent of the prevalence in the society, it therefore captures an absolute value judgment. C , however, captures a relative value judgment, and since an index cannot measure relative inequality from the perspectives of both health and ill-health, the importance of absolute inequality (and the value judgment) differs depending on the chosen perspective; C of health suggests that the same level of absolute inequality is more severe when the prevalence of health is low, while C of ill-health suggests that the inequality is more severe when the prevalence of health is high (i.e. when the prevalence of ill-health is low). As a consequence of the definition of the most unequal society, W combines these two perspectives by summing the magnitude of the relative inequalities in health and in ill-health. The value judgment underlying W , which we will refer to as mirror relativity, suggests (1) that the same level of absolute inequality is more severe for both high and low values of the prevalence, and (2) that inequalities increase (by the same magnitude as relative inequalities in ill-health) when all individuals' health increases proportionally. The formal definition of this value judgment illustrates that it reflects both relative inequality in health and relative inequality in ill-health.

In their latest exchange, Erreygers and Van Ourti (2011b), who argue in favor of an absolute index, and Wagstaff (2011b), who advocates a relative index, discard W because it is neither an absolute nor a relative index and it therefore behaves counterintuitively. However, neither side takes a stand on whether inequalities should be examined from the perspective of health or ill-health. Instead, Wagstaff (2011b) highlights conventions that have arisen for many commonly used variables; for example, there are conventions for using the fraction of immunized children rather than the fraction of unimmunized children and for using the fraction of infants dying before their first birthday rather than the fraction surviving.

The arbitrariness of these conventions is incompatible with basing the choice of index on a discussion of value judgments; why should we adopt totally different value judgments for immunization and for child mortality? Unless we have a priori reasons (or have at least had a serious discussion of these reasons)¹ to adopt the implicit value judgment of relative inequalities measured in either health or ill-health, neither of the relative indices are desirable. We claim instead that, as the counterintuitive behavior of W is a consequence of mirror relativity incorporating the two relative perspectives simultaneously, W represents an ethically defensible position and a reasonable alternative to the use of an absolute index (i.e. E) for binary variables.

In the next section, we present the methodological background of measuring health inequalities when the health variable is binary, and, as the concentration index approach does not meaningfully

measure inequalities in ordinal health variables (Erreygers, 2009a, p. 515), we provide a justification for using the concentration index approach with binary health measures. In Section 3, we examine the value judgments implicit in the indices, formally define mirror relativity, and illustrate that the technical advantages of E highlighted by Erreygers (2009a,b) are all consequences of the absolute value judgment. In Section 4, we empirically illustrate whether and when the choice of index affects comparisons between countries. These results confirm that the methodological discussion is truly important, and not just a matter of semantics. In Section 5, we present our conclusion and discuss the implications of the concept of mirror relativity.

2. Rank dependent inequality indices

C quantifies relative socioeconomic inequalities in a health variable, h_i , by calculating the cumulative percentage of h_i concentrated in a cumulative percentage of the population ranked by a socioeconomic variable (cf. Kakwani et al., 1997). C is equal to twice the area between the concentration curve and the line of equality (the 45 degree line). The related generalized concentration index (V) quantifies absolute inequalities and is equal to C multiplied by the prevalence, or mean, of the health variable μ_h (Clarke et al., 2002).

Erreygers (2009a) shows that C , V , W , and E all belong to the family of rank dependent indicators of health inequalities. Following Erreygers, we express the general form of this family of indicators as a normalized sum of weighted health levels:

$$I(h) = f(\mu_h, n) \sum_{i=1}^n z_i h_i \quad (1)$$

where $z_i = (n+1)/2 - \lambda_i$, $f(\mu_h, n) > 0$,² n is the number of individuals in a given population, and λ_i denotes the socioeconomic rank of the individual ranging from the richest ($\lambda_i = 1$) to the poorest ($\lambda_i = n$). When the health variable is binary, we may always use the vector $h = (h_1, h_2, \dots, h_n)$, which represents the health situation of the whole population, to construct a vector that represents the ill-health situation of the whole population, defined as shortfalls of health $(1-h) = (1-h_1, 1-h_2, \dots, 1-h_n)$. Thus, we may compute inequality indices of both health and ill-health, i.e. $I(h)$ and $I(1-h)$. As z_i takes on a positive value if individual i is rich (i.e. from the upper half of the income distribution) and a negative value if individual i is poor (i.e. from the lower half of the income distribution), a positive (negative) value of $I(h)$ suggests a pro-rich (pro-poor) concentration of health. Conversely, a positive (negative) value of $I(1-h)$ suggests a pro-poor (pro-rich) concentration of ill-health. As the function normalizing the weighted sum f is the only variation between the indices, it is this that determines the specific form and properties of the indices. We express C , V , W , and E for binary variables as:

$$C = f^C(\mu_h, n) \sum_{i=1}^n z_i h_i = \frac{2}{n^2 \mu_h} \sum_{i=1}^n z_i h_i \quad (2)$$

$$V = f^V(\mu_h, n) \sum_{i=1}^n z_i h_i = \frac{2}{n^2} \sum_{i=1}^n z_i h_i \quad (3)$$

$$W = f^W(\mu_h, n) \sum_{i=1}^n z_i h_i = \frac{2}{n^2(1-\mu_h)\mu_h} \sum_{i=1}^n z_i h_i \quad (4)$$

$$E = f^E(\mu_h, n) \sum_{i=1}^n z_i h_i = \frac{8}{n^2} \sum_{i=1}^n z_i h_i \quad (5)$$

¹ Recently, Allanson and Petrie (2012) initiated a discussion of the differences between value judgments that underly relative inequality measures of health and of ill-health.

² Compared with Erreygers (2009a), $f(\mu_h, n, a_h, b_h)$ is reduced to $f(\mu_h, n)$, as the lower (a_h) and upper (b_h) bounds of the binary health variable are equal to 0 and 1, respectively.

Table 1
Properties of the rank-dependent indices.

	Mirror	Transfer	Cardinal invariance	Level independence
<i>E</i>	✓	✓	✓	✓
<i>W</i>	✓	✓	✓ ^a	
<i>C</i>		✓		
<i>V</i>	✓	✓		✓

^a*C* satisfies cardinal invariance if modified as $C = [2/n^2(\mu_h - a_h)] \sum_{i=1}^n z_i h_i$ (Erreygers and Van Ourti, 2011a).

Moreover, Erreygers (2009a) shows that *E* is the only indicator within this family that satisfies the four properties considered by Erreygers to be desirable:

- (1) *transfer*: A small transfer of health from a richer (poorer) to a poorer (richer) individual translates into a pro-poor (pro-rich) change in the index.
- (2) *mirror*: The inequality indices of health and ill-health are mirror images of each other; that is, $I(h)$ is equal to the absolute value of $I(1 - h)$, but has the opposite sign.
- (3) *level independence*: An equal increment of health for all individuals does not affect the index; that is, the index is invariant to scalar addition even when the bounds of the variable are kept constant.
- (4) *cardinal invariance*: A linear transformation of the health variable, h_i , does not affect the value of the index; that is, the measured degree of inequalities is the same, irrespective of the cardinal scale of the health variable (e.g. $I[h]$ of body temperature would be the same whether measured in Celsius or Fahrenheit).

As summarized in Table 1, *V* satisfies all but cardinal invariance and *W* satisfies all but level independence, while *C* satisfies only transfer.³ However, Erreygers and Van Ourti (2011a) show that *C* is easily modified into a cardinal invariant index, which coincides with *C* for binary variables.⁴ Cardinal invariance is desirable as it means that the measured degree of inequality is the same irrespective of the two numeric values chosen to represent good and bad health (i.e. any dichotomous variable could be used). Unless there is a strong a priori reason for examining inequalities in the prevalence of either health or ill-health, the mirror condition is a desirable property when the health variable of interest is binary. The literature until recently (Allanson and Petrie, 2012) has been mute regarding such a reason, therefore imposing the mirror condition is reasonable, and *W* and *E* are left as the two possible choices (among the proposed indices). As the desirability of level independence, which constitutes the main difference between the two indices, is closely related to the value judgment inherent in the index, our further discussion below therefore focuses on the underlying value judgments in *E* and *W*. In order to illustrate the differences in the value judgment, we also continue to refer to *V*, *C*(*h*), and *C*($1 - h$).

2.1. A justification for binary variables

A binary variable is a specific representation of an ordinal or qualitative dichotomous variable. As such a scale is not usable with

³ As the normalization functions of *W* and *E*, respectively, in general are equal to $f^E(\mu_h, n, a_h, b_h) = 8/(n^2[b_h - a_h])$ and $f^W(\mu_h, n, a_h, b_h) = (2[b_h - a_h])/(n^2[b_h - \mu_h][\mu_h - a_h])$, $E = 4V$ holds for binary variables, but not in general, and *W* and *E* are invariant to any cardinal scaling of *h* (while *C* and *V* are not).

⁴ The modified $C = [2/n^2(\mu_h - a_h)] \sum_{i=1}^n z_i h_i$, where a_h is the lower bound of the health variable.

rank dependent indices (Erreygers, 2009a; Erreygers and Van Ourti, 2011a), another interpretation of the variable is needed.

Although an arbitrary dichotomous variable is at most ordinal, the binary representation has an absolute interpretation. The zero value of a binary variable, such as prevalence of any health condition, corresponds to a situation of complete absence, that is, zero indicates that the individual is not in the relevant health condition (cf. Roberts, 1979, pp. 64–65). Thus, the binary representation is compatible with the cumulative nature of *C*, as the successive addition of the “zeroes and ones” can intuitively be interpreted as the cumulative number of healthy or unhealthy individuals concentrated within a cumulative proportion of the population. Related to this understanding, one could consider the binary variable as projected onto a ratio-scaled measure of health bounded at 0 and 1 (h_i^*) by using the prevalence at an aggregated level of defined subgroups (e.g. deciles, percentiles) (cf. Erreygers and Van Ourti, 2011a). In fact, any rank-dependent index of h_i^* is asymptotically equal to an index of h_i .⁵

Interpreting the binary variable as a ratio-scaled proxy both facilitates the discussion of relative and absolute inequalities in Section 3 and provides a more intuitive understanding of level independence for binary variables. Instead of corresponding to invariance to a shift of all individuals from 0 to 1 – the only possible equal increment of a binary variable – level independence now corresponds to invariance to equal increments in prevalence across the quantiles.

3. Relative and absolute value judgments

For unbounded ratio-scaled variables, the notion of relative and absolute value judgments is clear-cut. Being invariant to an equal health increment but not to an equiproportionate change in the health variable is equivalent to being a measure of absolute inequalities (e.g. *V*). The opposite applies for measures of relative inequalities (e.g. *C*). It is clear from the discussion in Erreygers (2009a,b) and Wagstaff (2009) that it is not as straightforward for bounded health variables.

Although using h_i^* facilitates the discussion of relative and absolute inequalities, it does not address the issues induced by the bounds of the health variable. First, an increase in health is mirrored by a decrease in ill-health. Second, an equiproportionate change in health does not translate into an equiproportionate change in ill-health; as the health of everyone increases equiproportionately, the decrease in ill-health is smaller (in both absolute and relative terms) among the less healthy.⁶ Third, the bounds may act as constraints to (proportionally) equal transformations in the health variable. However, asymptotically it is always possible to define subgroups in such a way that marginal changes are feasible for almost all distributions of h_i^* .⁷ Nevertheless, one cannot

⁵ Let $I(h^*) = f(\mu_h, n) \sum_{i=1}^n z_i h_i$, where $h_i^* = \sum_{k=1}^l (h_i/k)$, l is equal to the number of observations in each subgroup, and l is the index of the subgroup. Then $I(h^*)$ will converge to $I(h)$ when either the number of observations or the number of groups increases. Increasing the number of groups is analogous to decreasing the number of observations within the group toward one. We are thankful to Guido Erreygers and Tom Van Ourti for comments on this specific issue.

⁶ In fact, an equiproportionate increase in health is translated into an equiproportionate decrease and an equal increment of ill-health. For example, let $h_i \beta = h_i(1 + \delta) = h_i + h_i \delta$, and let the shortfalls $(1 - h_i) = s_i$, then $h_i \delta = (1 - s_i) \delta = \delta - s_i \delta$. Thus, to both satisfy the mirror condition and remain invariant to equiproportionate changes, an index also has to be insensitive to equal increments (level independence).

⁷ Erreygers and Van Ourti (2011a) address this issue by redefining the concepts of relative and absolute to quasi-relative and quasi-absolute. Although we acknowledge the differences, we will in this paper still refer to these value judgments as absolute and relative.

directly translate the relative and absolute value judgments from unbounded variables to bounded ones.

The first two points suggest that unless the mirror condition is relaxed, an index of a bounded variable cannot be a pure measure of relative inequalities; such an index will always be sensitive to equiproportionate changes of health, and the response to equal increments will not be negative for all values of μ_h . While Erreygers and Van Ourti (2011a) therefore argue in favor of an absolute value judgment and Wagstaff (2011b) advocates relaxing the mirror condition, we present an alternative view.

The following sections reveal the underlying value judgment of the examined indices by exploring how, and why, equiproportionate changes and equal increments affect the indices. Above all, we provide a formal definition of the underlying value judgment of W . However, to understand these value judgments and the response of the indices, we first need to discuss how the indices define – and differ in their definition of – the most unequal society.

3.1. The most unequal society

C , W , and E all answer the question of how far a society is from the most unequal society by conditioning the level of actual absolute inequality on the absolute inequality in such a state. However, the definition of this state varies between the three indices.

To visualize the differences between W , E , and C , it is useful to express the indices in terms of a ratio between V of the observed state (i.e. actual level of absolute inequality) and V of the most unequal society according to the definition of the respective index ($V^{\max, I}$, where I refers to the specific index E , W , or C). As V is equal to twice the area between the line of equality and the generalized concentration curve – the cumulative population graphed against the cumulative amount of mean health μ_h – the numerator is always equal to area **I+II** in Fig. 1. The denominators reflect the indices' respective definitions of the most unequal society (see Appendix A1). According to C , the most unequal society is a state where all health is concentrated in the richest individual (i.e. $V^{\max, C}$ equals area **I+II+III+IV+VI**).

For a binary variable, such a state is not feasible unless there is only one unit of health to distribute within the population (i.e. only one individual is healthy). Wagstaff (2005) addresses exactly this issue when he normalizes C by $1/(1-\mu_h)$ (compare Eqs. (2) and (3)). Thus, in the most unequal society according to W , only the richest proportion of individuals is in good health, where this proportion always equals μ_h (i.e. $V^{\max, W}$ equals area **I+II+III+IV**). As both the lower left and the upper left corners of the area change with μ_h , the size of the denominator will depend on μ_h . Conversely, the society on which E conditions is independent of the prevalence in the society, and always corresponds to the richest 50% of individuals being in good health (i.e. $V^{\max, E}$ equals area **I+III+V**). The constant denominator of E corresponds to level independence. The denominators, and thus the values, of E and W coincide only when $\mu_h = 0.5$. As the definition of the most unequal society drives the differences between the two indices, it is crucial to keep the definitions in mind when evaluating the properties and the underlying value judgments of the indices.

3.2. The effect on indices of equal increments

Because V , a measure of absolute inequality and the numerator of E , W , and C , is insensitive to equal increments of health, the response of the indices to equal increments depends only on the response of the denominator. Evaluating the response of the denominator is equivalent to evaluating the derivative of the normalization function in Eqs. (1)–(5) with respect to μ_h (i.e. $\partial f[\mu_h, n]/\partial \mu_h$). The constant denominator (i.e. $\partial f^E[\mu_h, n]/\partial \mu_h = 0$)

confirms that E satisfies level independence and is insensitive to equal increments of prevalence across the quantiles – exactly as expected for an absolute index.⁸

As W satisfies the mirror condition, it cannot be a pure measure of relative inequality. In contrast to C 's unambiguously negative response to equal increments, W decreases for $\mu_h < 1/2$ and increases for $\mu_h > 1/2$; i.e. $\partial f^{C(h)}(\mu_h, n)/\partial \mu_h < 0$; $\partial f^W(\mu_h, n)/\partial \mu_h < 0$ for $\mu_h < 1/2$ and > 0 for $\mu_h > 1/2$.⁹ The translation of an equal increment in health to an increase in the index, despite the decrease in relative health differences, is the opposite of what is expected from a relative inequality index. This counterintuitive behavior occurs because prevalence is a bounded variable, and when health increases from, for example, 0.7 to 0.8, there is a simultaneous decrease in ill-health in the same variable from 0.3 to 0.2. Imposing the mirror condition implies that the response to changes in health must equal the absolute value of the response to changes in ill-health, but with opposite sign. Consequently, the marginal response of W to an equal increment, $\partial f^W(\mu_h, n)/\partial \mu_h$, for $\mu_h < 1/2$ is the negative mirror of the same function for $\mu_h > 1/2$. Rewriting the normalization function of W , $f^W(\mu_h, n)$, as the sum of the normalization function of $C(h)$ and $C(1-h)$, i.e. the sum of $f^{C(h)}(\mu_h, n)$ and $f^{C(1-h)}(1-\mu_h, n)$:

$$\begin{aligned} f^W(\mu_h, n) &= \frac{2}{n^2(1-\mu_h)\mu_h} = \frac{2}{n^2\mu_h} + \frac{2}{n^2(1-\mu_h)} \\ &= f^{C(h)}(\mu_h, n) + f^{C(1-h)}(1-\mu_h, n) \end{aligned} \quad (6)$$

implies that the marginal response of W equals the sum of the marginal response, with respect to health, of $C(h)$ and $C(1-h)$:

$$\begin{aligned} \frac{\partial f^W(\mu_h, n)}{\partial \mu_h} &= \frac{(4\mu_h - 2)}{n^2(\mu_h - \mu_h^2)^2} = -\frac{2}{n^2\mu_h^2} + \frac{2}{n^2(1-\mu_h)^2} \\ &= \frac{\partial f^{C(h)}(\mu_h, n)}{\partial \mu_h} + \frac{\partial f^{C(1-h)}(1-\mu_h, n)}{\partial \mu_h} \end{aligned} \quad (7)$$

Thus, the change in the degree of inequality due to an equal increment in health equals the sum of the changes in relative inequality in health, $C(h)$, and relative inequality in ill-health $C(1-h)$. Eq. (7) also reveals that the marginal response of W always has the same sign as (and a similar shape to) the marginal response of the C that represents the variable, health or ill-health, with the lowest mean and largest change in inequality. That is, if $\mu_h < 1/2$ then $|\partial f^{C(h)}(\mu_h, n)/\partial \mu_h| > |\partial f^{C(1-h)}(1-\mu_h, n)/\partial \mu_h|$, and $\partial f^W(\mu_h, n)/\partial \mu_h$ has the same sign as $\partial f^{C(h)}(\mu_h, n)/\partial \mu_h$, and vice versa.

Fig. 2 further demonstrates this relationship and the underlying value judgments of the indices by graphing the weights of absolute inequalities – the normalization function f or equivalently the inverse of the most unequal society $1/V^{\max, I}$ (see Appendix A1) – as a function of μ_h for W , E , $C(h)$, and $C(1-h)$. The weight of $C(h)$ suggests that the severity of a given level of absolute inequality rises at an increasing rate as the total amount of health in the population falls, while the weight of $C(1-h)$ suggests the opposite, that the severity rises increasingly as the total amount of health rises. Thus, with a certain level of absolute inequality and a negative relationship between health and socioeconomic status (as is commonly the case), using $C(h)$ or $C(1-h)$ implies two opposing positions. $C(h)$ represents a position that considers a society where the total amount of health is low and the few healthy individuals are mostly concentrated in the richest percentiles to be more unequal than a society where the total amount of health is high and the few

⁸ Level independence corresponds to Erreygers and Van Ourti's (2011a) definition of a quasi-absolute measure for bounded variables.

⁹ $\partial f^{C(h)}(\mu_h, n)/\partial \mu_h = -2/(n^2\mu_h^2) < 0$; $\partial f^W(\mu_h, n)/\partial \mu_h = (4\mu_h - 2)/(n^2(\mu_h - \mu_h^2)^2) = 0$ if $\mu_h = 0$; > 0 for $\mu_h > 1/2$; < 0 for $\mu_h < 1/2$;

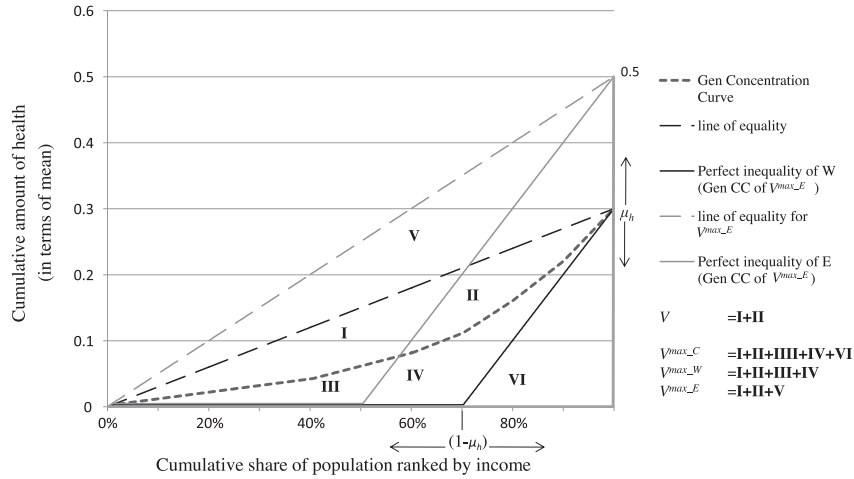


Fig. 1. The most unequal society. *Note:* If the health variable is ill-health rather than health, then the definition of the most unequal society is reversed; that is, the ill-health is concentrated in the richest individual (C), the richest share of the individuals are in bad health (W), and the richest 50% of the individuals are in bad health (E). However, the area representing these reversed inequalities is as large as the area between the line of equality and the imaginary line of perfect inequality, representing a state where the poorest individuals are in bad health. As these areas would be above the line of equality, using such a definition would require that the absolute inequalities are conditioned on the absolute value of $V^{max,J}$.

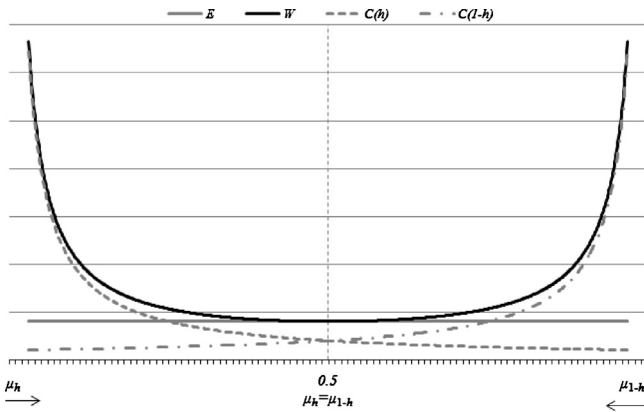


Fig. 2. f or $1/V^{max,J}$ of W , E , $C(h)$, and $C(1-h)$. *Note:* The functions are shown for any arbitrary value of n . $\partial f^W(\mu_h, n)/\partial \mu_h$ (the derivative of black line) has the same sign (negative) as $\partial f^{C(1-h)}(1-\mu_h, n)/\partial \mu_h$ (the derivative of dash) as long as $\mu_h < 1/2$, whereas for $\mu_h > 1/2$, $\partial f^W(\mu_h, n)/\partial \mu_h$ has the same sign (positive) as $\partial f^{C(h)}(\mu_h, n)/\partial \mu_h$ (the derivative of dash-dot).

sick (unhealthy) individuals are mostly concentrated in the poorest percentiles. $C(1-h)$ represents the opposite position. By weighting the degree of absolute inequality by the sum of $f^{C(h)}(\mu_h, n)$ and $f^{C(1-h)}(1-\mu_h, n)$ (i.e. a strictly convex function that is symmetric around 0.5),¹⁰ W combines the two perspectives, suggesting that the same level of absolute inequality is more severe for extreme values of the prevalence, that is, either high or low. As W reflects the perspective – health or ill-health – with the lowest prevalence (and the highest level of relative inequality) the behavior may not be as counterintuitive as it first appears (compare Erreygers and Van Ourti, 2011a). We develop and formalize this argument after discussing the response of the indices to equiproportionate changes in health in the following section.

3.3. Linearity and the effect on indices of equiproportionate changes

In line with what is expected from an absolute index, E increases as a response to an equiproportionate increase in health (note that when evaluating the response to an equiproportionate change in health, we need to consider the ratio-scaled projection of h , i.e. h^*). Erreygers (2009b) further highlights the linearity of E .

Linearity: A reduction of every individual's health from h_i^* to βh_i^* , where $0 \leq \beta < 1$, implies that $\beta I(h^*) = I(\beta h^*)$.

A rank-dependent index satisfies linearity only if it is level independent.¹¹ Provided that the transformation is feasible, linearity also means that if the prevalence is doubled in every decile, the measured degree of inequality is doubled as well. Because the numerator of the indices in Fig. 1 (absolute inequalities) satisfies linearity, E satisfies this property with its constant denominator.

W 's response is more complex than both the easily interpreted linearity of E and the invariance of $C(h^*)$. An increase in health from h^* to βh^* implies that V increases by β , while the denominator of W , $V^{max,W}$, increases by less than β , and thus W increases as the society moves further away from the most unequal state. In fact, the response of W is increasingly positive in β , i.e. $\partial W(\beta h^*)/\partial \beta > 0$; $\partial^2 W(\beta h^*)/\partial \beta^2 > 0$. Rewriting W as:

$$W(h) = \frac{1}{(1-\mu_h)\mu_h} V(h) = \left(\frac{1}{\mu_h} + \frac{1}{(1+\mu_h)} \right) V(h) = C(h) - C(1-h), \tag{8}$$

shows that W equals the sum of the measures of relative inequality in health and ill-health¹² and illustrates that the increase of inequality according to W is always of the same magnitude as the increase according to $C(1-h^*)$, i.e. $\partial W(\beta h^*)/\partial \beta = -\partial C(1-\beta h^*)/\partial \beta$ (remember that the interpretation of the sign of $I[1-h]$ is the reverse of $I[h]$). Thus, by incorporating the two relative perspectives in the same index, W takes into account that the change in ill-health

¹⁰ That is, $\partial^2 f^W(\mu_h, n)/\partial \mu_h^2 = (4(3\mu_h^2 - 3\mu_h + 1))/(n^2(\mu_h - \mu_h^2)^3) > 0$ for $0 < \mu_h < 1$.

¹¹ $I(\beta h^*) = f(\mu_h, \beta, n) \sum_{i=1}^n z_i \beta h_i^* = f(\mu_h, \beta, n) \beta \sum_{i=1}^n z_i h_i^* = \beta f(\mu_h, n) \sum_{i=1}^n z_i h_i^* = \beta I(h^*)$ if and only if $f(\mu_h, \beta, n) = f(\mu_h, n)$, which is equivalent to level independence or $\partial f(\mu_h, n)/\partial \mu_h = 0$.

¹² Observe that the varying part of the normalization function (i.e. the inverse of the most unequal society) of W is both the product and the sum of the varying part of the normalization function of $C(h)$ and $C(1-h)$.

that mirrors the equiproportional change in health becomes more disproportional as β increases. That is, while the relative inequality in h is constant, the relative inequality in $(1 - h)$ accelerates. However, as the part of W that reflects relative inequality in h is unaffected, the relative change in $W(h^*)$ is still smaller than the relative change in $C(1 - h^*)$:

$$\frac{C(1 - \beta h^*)}{C(1 - h^*)} = \frac{V(1 - \beta h^*)/(1 - \beta \mu_h)}{V(1 - h^*)/(1 - \mu_h)} = \beta \frac{(1 - \mu_h)}{(1 - \beta \mu_h)} > \frac{(1 - \mu_h)}{(1 - \mu_h)} = \frac{W(\beta h^*)}{W(h^*)} \text{ if } \beta > 1 \quad (9)$$

In the next section, we further develop this argument and formally define the value judgment that underlies W .

3.4. Mirror relativity

The last two sections suggest that by normalizing C to the most unequal society possible (given the prevalence), W takes both relative inequality in health and relative inequality in ill-health into account in one and the same index. To simultaneously consider the health and ill-health distributions in one index that satisfies the mirror condition, without necessarily reverting to a measure of absolute inequality, is in line with the framework suggested in a recent paper by Lasso de la Vega and Aristondo (2012).¹³ One way of combining the two relative value judgments is a linear combination of the two relative indices, $C(h)$ and $C(1 - h)$, i.e. $I(h) = aC(h) + bC(1 - h)$, where a and b are constants. Imposing the mirror condition further implies that $a = -b$ must hold.¹⁴ Since the interpretation of the sign of $C(1 - h)$ is the reverse of $C(h)$, such an index sums the magnitude of relative inequality in health and the magnitude of relative inequality in ill-health into one index. We refer to this normative attribute as *mirror relativity*.

Mirror relativity: $I(h)$ is mirror relative index if

$$I(h) = B[C(h) - C(1 - h)] \quad (10)$$

where B is a positive constant.

The class of mirror relative index consists only of multiples of W .¹⁵ *Mirror relativity* implies that (1) inequality in health increases by the same magnitude as relative inequality in ill-health, i.e. any scalar multiple of $C(1 - h)$, when the health of everyone increases proportionally, and (2) the degree of absolute inequality is weighted by a U-shaped function (a strictly convex function symmetric around 0.5).

Proposition 1. *A rank dependent index $I(h)$ is mirror relative if and only if*

- (1) *For an equiproportionate change in health from h^* to βh^* , $I(h^*)$ changes by the same magnitude as a rank-dependent index that is relative from the perspective of ill-health, i.e.*

¹³ Lasso de la Vega and Aristondo (2012) propose a unified framework where the achievement and the shortfall distributions can be jointly analyzed, without restricting the indices to an absolute value judgment. Although their framework considers univariate indices, it is easily transferable to the family of bivariate rank-dependent indices. Thus, our argument in favor of mirror relativity follows in the line of their reasoning and the class of mirror relative indices (corresponding to a scalar multiple of the arithmetic mean of $C[h]$ and $-C[1 - h]$) is a special case ($r = 1$) of their suggested r -indicators translated to the Concentration Index.

¹⁴ Let $I(h) = aC(h) + bC(1 - h)$, where a and b are constants ($f(\mu_h, n) > 0$ further requires $a > 0$ and $b < 0$), satisfy the mirror condition, i.e. $I(h) = -I(1 - h)$. Rewriting $I(h) = aC(h) + bC(1 - h) = -(aC(1 - h) + bC(h)) = -I(1 - h)$, as $[aV(h)/(\mu_h) - bV(1 - h)/(1 - \mu_h)] = [aV(1 - h)/(1 - \mu_h) - bV(h)/\mu_h]$ implies $aV(h)/\mu_h(1 - \mu_h) = -[bV(h)/\mu_h(1 - \mu_h)]$, which reduces to $a = -b$.

¹⁵ W (i.e. $B = 1$) is the only index in this class that is bounded between 0 and 1.

$$\frac{\partial W(\beta h^*)}{\partial \beta} = - \frac{\partial \left(\frac{A}{(1 - \beta \mu_h)} \sum_{i=1}^n z_i(1 - \beta h_i^*) \right)}{\partial \beta} \text{ for all admissible } \beta, \text{ where } A \text{ is a positive constant.}$$

- (2) *The normalization function f is a strictly convex function in μ_h and symmetric around $\mu_h = 0.5$.*

Proof. Proof in Appendix A2.

Thus, mirror relativity reflects the disproportionality of the decrease in ill-health that occurs when the health of everyone increases proportionally. The larger the increase in health, the more disproportional the change in ill-health. Mirror relativity also reflects the fact that for an equal increment of health that leaves all absolute differences the same, relative inequality in health increases while relative inequality in ill-health decreases. A mirror-relative index decreases as long as the decrease in relative inequality in health exceeds the increase in relative inequality in ill-health.

As this value judgment cannot be expressed by a simple equivalence criterion (as an absolute or relative value judgment), we will use the elasticity of the normalization function (or the most unequal society),

$$\varepsilon(\mu_h) = \frac{\partial f(\mu_h, n)}{\partial \mu_h} \frac{\mu_h}{f(\mu_h, n)} \quad (11)$$

to further relate mirror relativity to the commonly known value judgments. Erreygers and Van Ourti (2011a) use $\varepsilon(\mu_h)$ to develop measures of an index's sensitivity to relative and absolute inequality in order to define different classes of indices. They define the weight an index gives to relative inequality as $-\varepsilon(\mu_h)$ and the weight it gives to absolute inequality as $1 + \varepsilon(\mu_h)$. Thus, indices for which $-\varepsilon(\mu_h) = 1$ are defined as relative, i.e. $\varepsilon(\mu_h) = -1$; indices for which $1 + \varepsilon(\mu_h) = 1$ are defined as absolute, i.e. $\varepsilon(\mu_h) = 0$; indices for which both weights are positive are called *mixed inequality* indices, i.e. $-1 < \varepsilon(\mu_h) < 0$; and indices for which the relative inequality weight is negative and the absolute inequality weight is greater than 1 are called *inverse-relative* indices, i.e. $\varepsilon(\mu_h) > 0$. An inverse-relative index increases in magnitude for an equal increment, although that also means that all relative differences decrease. For example, an index that is relative from the perspective of ill-health, i.e. any multiple of $C(1 - h)$, is inverse-relative from the perspective of health. To show this property we introduce a similar but modified elasticity concept; the elasticity of the normalization function with respect to the inverse perspective $(1 - \mu_h)$.

$$\eta(1 - \mu_h) = \frac{\partial f(\mu_h, n)}{\partial (1 - \mu_h)} \frac{(1 - \mu_h)}{f(\mu_h, n)} \quad (12)$$

To obtain the weight $C(1 - h)$ puts on absolute inequalities and relative inequalities in health, let $\eta(\mu_h)$ replace $\varepsilon(\mu_h)$ in the definitions above. As the elasticity of the normalization function of $C(1 - h)$ with respect to μ_h equals $\eta^{C(1-h)}(\mu_h) = [\partial f(1 - \mu_h, n)/\mu_h][\mu_h/f(1 - \mu_h, n)] = \mu_h/(1 - \mu_h)$ and $\eta^{C(1-h)}(\mu_h) > 0$ holds for all μ_h , $C(1 - h)$ is inverse relative from the perspective of health.

The elasticity of the normalization function of W (or any mirror relative index) equals $\varepsilon^W(\mu_h) = (2\mu_h - 1)/(1 - \mu_h)$ and implies that W is a mixed inequality index for $\mu_h < 0.5$ and an inverse-relative index for $\mu_h > 0.5$. Thus, inverse-relativity is the formal definition of the counterintuitive behavior that we previously discussed. Rewriting the elasticity of W as the sum of $\varepsilon^{C(h)}(\mu_h)$ and $\eta^{C(1-h)}(\mu_h)$ (i.e. the elasticities with respect to health of $C[h]$ and $C[1 - h]$) illustrates the notion of mirror relativity; a mirror relative index is a compromise that takes into account, and is bounded between, the two relative value judgment.

Proposition 2. A rank dependent index $I(h)$ is mirror relative if and only if:

$$\varepsilon(\mu_h) = \frac{2\mu_h - 1}{1 - \mu_h} = -1 + \frac{\mu_h}{1 - \mu_h} = \varepsilon^{C(h)}(\mu_h) + \eta^{C(1-h)}(\mu_h) \quad (13)$$

that is, the elasticity of the normalization function equals the sum of the elasticities, with respect to h , of the normalization functions for $C(h)$ and $C(1-h)$.

Proof. Proof in Appendix A2.

Thus, the weight a mirror relative index puts on relative inequalities in health (i.e. $-\varepsilon[\mu_h]$) equals the sum of the weights that $C(h)$ and $C(1-h)$ puts on relative inequalities in health. The elasticity formula in Eq. (13) illustrates that the larger the μ_h , the larger the influence from relative inequalities in ill-health in relation to the influence from relative inequalities in health and vice versa. As μ_h tends to one of its bounds, the value judgment a mirror relative index embodies tends to one of the two relative judgments. Eq. (13) further illustrates that the mentioned counterintuitive behavior is a consequence of W taking both relative perspectives into account. As the first part of Eq. (13) equals $\varepsilon^{C(h)}(\mu_h) = -1$ (i.e. relative inequalities in h are unchanged), the (increasingly) positive response to an equiproportionate change is due solely to the (increasingly) positive second part $\eta^{C(1-h)}(\mu_h) > 0$ (i.e. the relative inequalities in $1-h$ increase). As the influence of relative inequalities in ill-health exceeds the influence of relative inequalities in health (i.e. $\eta^{C(1-h)}[\mu_h] > -\varepsilon^{C(h)}[\mu_h]$) for $\mu_h > 1/2$, a mirror relative index is inverse-relative for this span.

At first sight it may seem difficult to argue in favor of an inverse-relative index (compare Erreygers and Van Ourti, 2011a). However, for binary variables inequalities may be evaluated from both the perspective of health and the perspective of ill-health. Although $C(h)$ is relative from the perspective of health, it is by definition inverse-relative from the perspective of ill-health and vice versa for $C(1-h)$. While a mirror relative index is inverse-relative for $\mu_h > 1/2$ from the perspective of health, it is mixed from the perspective of ill-health and vice versa for $\mu_h < 1/2$.

As noted in the introduction, there is no consensus on the criteria for choosing the perspective from which health inequalities should be evaluated, and therefore no clear reason why we should apply different relative value judgments to immunization and child mortality. A position that argues against a mirror relative index simply because of its inverse-relativity, while at the same time approving both of the relative indices equally (i.e. allowing the arbitrary choice of one or the other), we find difficult to support. Instead, referring to the previous discussion, we argue that a mirror relative index (i.e. W) is a plausible compromise between the two relative perspectives and that such an index is a reasonable alternative to an absolute index (i.e. E) for binary variables.

3.5. Critique against mirror relativity

Erreygers (2009a,b) puts forward two additional properties – *monotonicity* and *convergence* – as arguments for preferring E over W . E satisfies both these properties while W does not. However, the following section illustrates that these properties are a result of the definition of the most unequal society (i.e. the constant $V^{max.E}$ in Fig. 1) and are only desirable if we want to apply an absolute value judgment.

3.5.1. Individual changes and monotonicity

When considering individual changes in health, level independence again constitutes the crucial difference between the indices. Erreygers (2009a) shows that level independence implies monotonicity.

Monotonicity: If an individual from the upper half of the income distribution enters a condition of good health (a pro-rich health improvement), then $I(h)$ increases (a pro-rich change).

As the denominator, the most unequal society, of E is constant, an individual health change modifies only the numerator of the index, and the change in E depends only on the socioeconomic rank of the individual who is changing health state. As a result, a pro-rich health improvement translates into a pro-rich change in E .

Because W is not level independent, its denominator is not constant and it does not satisfy monotonicity; in addition to the individual's socioeconomic rank, the sign (and size) of the change in W also depends on the initial prevalence, μ_h , and the initial level of absolute inequalities (see Appendix A3). Therefore, a pro-rich health improvement does not necessarily translate into a pro-rich change in W .

Consequently, Erreygers (2009a, p. 508) criticizes W for producing artificial and counterintuitive results. For example, if the richest 10% of a population is in good health, then an additional rich individual (not ranked in the 11th percentile) entering a state of good health implies that E increases but W decreases. This health improvement is, according to Erreygers, obviously pro-rich but translates into a pro-poor change of W .

However, the *non-monotonicity* arises from the value judgments underlying W (and C). Most important, the change of the health distribution in the example above is a movement away from the most unequal society, as defined by W as a society in which only the individuals at the top of the income distribution are in good health. Likewise, the monotonicity of E arises from the definition of the most unequal society as a society in which only the richest 50% of the population is in good health, because in this case an additional individual from the upper half of the income distribution entering a state of good health is always a movement toward such a state. Thus, as Wagstaff (2009) points out, the desirability of monotonicity depends on the question behind the index and the value judgment one wants to impose.

3.5.2. The convergence property

Erreygers (2009b) states that if the health of every individual is gradually reduced to zero, that is, if the society approaches a state of perfect equality, then the measured degree of inequality should also decrease to zero.

Convergence: If every individual's health decreases from h_i^* to βh_i^* , where $\beta < 1$, then $I(h^*)$ converges to zero, i.e. $\lim_{\beta \rightarrow 0} I(\beta h^*) = 0$.

Any absolute (level independent) rank-dependent index, including E , satisfies this property.¹⁶ By contrast, neither a relative index, which by definition is invariant to an equiproportional change, nor a mirror relative index converges to zero. Mirror relativity instead implies a different convergence concept, i.e. $\lim_{\beta \rightarrow 0} I(\beta h^*) = C(h^*)$. That a mirror relative index converges to a relative judgment in the lower limit may be intuitively illustrated using the elasticity formula in Eq. (13), i.e. $\varepsilon^W(\mu_h) = \varepsilon^{C(h)}(\mu_h) + \eta^{C(1-h)}(\mu_h)$. As $\eta^{C(1-h)}(\mu_h)$ approaches zero when μ_h approaches zero (i.e. $C(1-h)$ ceases to put any weight on relative inequalities in health), $\varepsilon^W(\mu_h)$ approaches $\varepsilon^{C(h)}(\mu_h) = -1$ (i.e. as the relative inequalities in ill-health converge to zero, W only considers relative inequalities in health).

¹⁶ In Fig. 1, $V = \text{I} + \text{II}$ – the absolute inequality or the numerator of the index – gradually decreases toward zero as μ_h decreases. As the denominator of E ($V^{max.E} = \text{I} + \text{II} + \text{V}$) is constant, the ratio will converge to zero. Fig. 1 further illustrates that the denominator of W ($\text{I} + \text{II} + \text{III} + \text{IV}$) converges to that of C ($\text{I} + \text{II} + \text{III} + \text{IV} + \text{VI}$) as area VI approaches zero.

Erreygers (2009b, p. 523) points out this lack of convergence as the major shortcoming of W , because it means that W may exaggerate the measured degree of inequality when μ_h is approaching its limits. However, we claim that W does not exaggerate the level of inequalities for low values of μ_h but measures relative rather than absolute inequalities. Thus, if an index considers both relative inequalities in health and relative inequalities in ill-health, the convergence property proposed by Erreygers (2009b) is not necessarily an appealing property.

4. Empirical analysis

4.1. Data

Empirically, we examine how the choice of index affects comparisons between countries using nine binary indicators of bad health from the second wave of the European Survey of Health, Ageing and Retirement (SHARE).¹⁷ The nine indicators are: having diabetes, having cancer, being a daily smoker, having a long-term illness, having more than two limitations in daily life, having more than two chronic diseases, and three measures of bad self-assessed health (SAH). The three SAH measures are from the same reported ordinal scale, but are coded at different cut-off points.¹⁸ As the socioeconomic ranking variable, we use equivalent income on a household level.¹⁹

To examine whether the choice of index affects comparisons between countries, we compute correlation coefficients of the indices. However, what really matters is whether the choice of index affects the rank of the countries; for example, if health is more equally distributed in country A than in country B according to W , but the reverse pattern emerges according to E . Therefore, we compute Spearman's and Kendall's rank correlation coefficients.

4.2. Results

The indices in Table 2 and the rank correlation coefficients in Table 3 show that the choice of index may affect the ranking and thus in turn affect the outcome of a comparison. The ranking of the 14 countries are different for E and W for all nine health indicators, albeit to differing extents.

W and E , as well as their definitions of the most unequal society, coincide when $\mu_{1-h} = \mu_h = 1/2$ and diverge for large and small values of μ_{1-h} (see Eqs. (4) and (5)). Because E is level independent while W is not, the ranking based on the two indices will be different when μ_h varies between contexts. The empirical findings presented in Tables 2 and 3 confirm these claims. For health variables in which μ_{1-h} is close to $1/2$ for all countries (e.g. long-term illness) the rankings based on the two indices practically coincide. By contrast, if μ_{1-h} varies substantially across the countries (e.g. SAH 3), the rank correlation is low. Although the number of observations is small, we can also easily verify this twofold conclusion by running a regression of the rank correlation coefficients on the mean of the prevalence $\bar{\mu}_{1-h}$, the squared mean of the prevalence $\bar{\mu}_{1-h}^2$, and the standard deviations of the prevalence. The results in Table 4 first show that there is a U-shaped association between the mean of the prevalence $\bar{\mu}_{1-h}$ and the rank correlation coefficient;

Table 2
Empirical comparison of W and E in 14 countries.

	μ_{1-h}	W	E	Rank W	Rank E
Cancer					
Austria	0.02	0.30	0.02	1	3
Germany	0.04	-0.18	-0.03	13	14
Sweden	0.07	-0.09	-0.02	10	11
Netherlands	0.03	-0.01	0.00	9	9
Spain	0.02	0.13	0.01	5	4
Italy	0.03	0.24	0.03	2	2
France	0.04	0.20	0.03	3	1
Denmark	0.07	-0.11	-0.03	11	13
Greece	0.01	0.17	0.01	4	5
Switzerland	0.04	0.00	0.00	7	7
Belgium	0.03	-0.19	-0.03	14	12
Czech Rep.	0.05	0.03	0.01	6	6
Poland	0.03	-0.01	0.00	8	8
Ireland	0.05	-0.11	-0.02	12	10
Diabetes					
Austria	0.11	0.02	0.01	3	3
Germany	0.15	-0.14	-0.07	13	14
Sweden	0.09	-0.13	-0.04	11	7
Netherlands	0.09	-0.07	-0.02	5	5
Spain	0.15	-0.12	-0.06	8	12
Italy	0.12	-0.04	-0.02	4	4
France	0.10	-0.15	-0.05	14	11
Denmark	0.08	-0.12	-0.04	7	6
Greece	0.13	-0.12	-0.05	6	10
Switzerland	0.06	0.07	0.02	1	2
Belgium	0.10	-0.12	-0.04	9	8
Czech Rep.	0.14	-0.13	-0.06	10	13
Poland	0.11	0.06	0.02	2	1
Ireland	0.10	-0.14	-0.05	12	9
Long-term illness					
Austria	0.45	-0.07	-0.06	3	3
Germany	0.59	-0.11	-0.10	7	7
Sweden	0.54	-0.20	-0.20	13	14
Netherlands	0.44	-0.07	-0.07	4	5
Spain	0.56	-0.15	-0.15	10	10
Italy	0.42	-0.07	-0.07	5	4
France	0.50	-0.12	-0.12	8	8
Denmark	0.48	-0.16	-0.16	11	11
Greece	0.37	-0.21	-0.20	14	13
Switzerland	0.36	-0.08	-0.07	6	6
Belgium	0.44	-0.12	-0.12	9	9
Czech Rep.	0.55	-0.20	-0.19	12	12
Poland	0.66	-0.06	-0.05	2	2
Ireland	0.42	-0.02	-0.02	1	1
SAH 1 (poor)					
Austria	0.07	-0.06	-0.02	2	2
Germany	0.11	-0.27	-0.10	11	12
Sweden	0.07	-0.33	-0.09	14	11
Netherlands	0.05	-0.30	-0.05	12	7
Spain	0.14	-0.16	-0.07	7	9
Italy	0.13	-0.10	-0.05	5	5
France	0.09	-0.19	-0.07	9	8
Denmark	0.06	-0.33	-0.08	13	10
Greece	0.06	-0.09	-0.02	3	3
Switzerland	0.03	0.02	0.00	1	1
Belgium	0.07	-0.10	-0.02	4	4
Czech Rep.	0.14	-0.25	-0.12	10	14
Poland	0.34	-0.12	-0.10	6	13
Ireland	0.07	-0.19	-0.05	8	6
SAH 2 (less than good)					
Austria	0.31	-0.22	-0.19	8	7
Germany	0.41	-0.24	-0.23	10	13
Sweden	0.30	-0.27	-0.22	13	10
Netherlands	0.29	-0.15	-0.13	4	4
Spain	0.46	-0.20	-0.20	6	8
Italy	0.44	-0.11	-0.11	2	3
France	0.36	-0.23	-0.21	9	9
Denmark	0.25	-0.31	-0.23	14	12
Greece	0.30	-0.27	-0.23	12	11
Switzerland	0.18	-0.12	-0.07	3	1
Belgium	0.30	-0.22	-0.18	7	6

¹⁷ The settings include Austria, Germany, Sweden, Netherlands, Spain, Italy, France, Denmark, Greece, Switzerland, Belgium, Czech Republic, Poland, and Ireland.

¹⁸ The ordinal scale is (1) excellent, (2) very good, (3) good, (4) fair, and (5) poor. SAH 1 is equal to one if the respondent has reported having poor health (5). SAH 2 corresponds to less than good health (4 or 5), and SAH 3 corresponds to less than very good health (3, 4, or 5).

¹⁹ Reported household income in the last month divided by the square root of the household size.

Table 2 (Continued)

	μ_{1-h}	W	E	Rank W	Rank E
Czech Rep.	0.46	-0.26	-0.26	11	14
Poland	0.63	-0.10	-0.09	1	2
Ireland	0.25	-0.20	-0.15	5	5
SAH 3 (less than very good)					
Austria	0.73	-0.19	-0.15	6	6
Germany	0.81	-0.24	-0.15	11	4
Sweden	0.59	-0.27	-0.26	12	13
Netherlands	0.72	-0.20	-0.16	7	8
Spain	0.87	-0.19	-0.09	4	3
Italy	0.81	-0.13	-0.08	1	2
France	0.79	-0.22	-0.15	9	5
Denmark	0.49	-0.28	-0.28	13	14
Greece	0.63	-0.24	-0.22	10	12
Switzerland	0.54	-0.18	-0.18	3	10
Belgium	0.71	-0.19	-0.16	5	7
Czech Rep.	0.82	-0.32	-0.18	14	11
Poland	0.93	-0.21	-0.06	8	1
Ireland	0.54	-0.16	-0.16	2	9
Limitations					
Austria	0.08	-0.04	-0.01	2	2
Germany	0.06	-0.18	-0.04	6	6
Sweden	0.04	-0.38	-0.06	13	10
Netherlands	0.03	-0.28	-0.04	10	5
Spain	0.07	-0.28	-0.08	11	12
Italy	0.08	0.01	0.00	1	1
France	0.07	-0.21	-0.06	8	9
Denmark	0.06	-0.41	-0.09	14	14
Greece	0.06	-0.15	-0.04	4	4
Switzerland	0.02	-0.27	-0.02	9	3
Belgium	0.09	-0.17	-0.05	5	7
Czech Rep.	0.07	-0.30	-0.08	12	13
Poland	0.17	-0.10	-0.05	3	8
Ireland	0.09	-0.21	-0.07	7	11
Chronic disease					
Austria	0.20	-0.05	-0.03	2	3
Germany	0.21	-0.15	-0.10	7	8
Sweden	0.21	-0.29	-0.20	13	13
Netherlands	0.14	-0.15	-0.07	9	5
Spain	0.23	-0.15	-0.11	8	9
Italy	0.27	-0.10	-0.08	5	6
France	0.18	-0.24	-0.14	11	10
Denmark	0.23	-0.32	-0.23	14	14
Greece	0.21	-0.28	-0.18	12	12
Switzerland	0.09	-0.09	-0.03	4	2
Belgium	0.22	-0.13	-0.09	6	7
Czech Rep.	0.25	-0.21	-0.16	10	11
Poland	0.31	-0.03	-0.03	1	1
Ireland	0.22	-0.09	-0.06	3	4
Smoking					
Austria	0.14	0.06	0.03	5	5
Germany	0.18	0.00	0.00	8	8
Sweden	0.17	-0.04	-0.02	13	13
Netherlands	0.25	-0.12	-0.09	14	14
Spain	0.16	0.22	0.12	1	2
Italy	0.17	0.09	0.05	4	4
France	0.15	0.03	0.02	7	7
Denmark	0.28	-0.01	-0.01	10	10
Greece	0.29	0.16	0.13	2	1
Switzerland	0.18	-0.03	-0.02	12	12
Belgium	0.19	0.05	0.03	6	6
Czech Rep.	0.21	0.14	0.09	3	3
Poland	0.27	-0.02	-0.02	11	11
Ireland	0.19	0.00	0.00	9	9

that is, the ranking according to W and the ranking according to E diverge for low and high values of $\tilde{\mu}_{1-h}$. Second, the results also confirm that the larger the variation in prevalence, the smaller the rank correlation coefficient.

Table 3

Rank correlations coefficients.

Health indicator	Spearman	Kendall	Pearson	SD ^a of μ_{1-h}	$\tilde{\mu}_{1-h}$
Cancer	0.947	0.824	0.940	0.017	0.039
Diabetes	0.824	0.692	0.959	0.025	0.110
Long-term illness	0.991	0.956	0.998	0.085	0.484
SAH 1	0.741	0.604	0.705	0.075	0.101
SAH 2	0.903	0.758	0.931	0.117	0.352
SAH 3	0.486	0.319	0.617	0.135	0.713
Limitation	0.741	0.582	0.742	0.034	0.070
Chronic disease	0.938	0.846	0.975	0.053	0.213
Smoking	0.996	0.978	0.982	0.049	0.202

^a SD: standard deviation.

Table 4

Regression results.

	SD ^a of $\tilde{\mu}_{1-h}$	$\tilde{\mu}_{1-h}$	$\tilde{\mu}_{1-h}$
Spearman	-3.06	2.40	-3.07
Kendall	-4.50	3.28	-3.96

^a SD: standard deviation.

5. Conclusion

To appropriately measure inequalities of binary health variables it is important to pay attention to both the normative and the technical aspects of an inequality index, but it is also important to keep the two aspects separate. Even though C , V , W , and E all belong to the family of rank-dependent indicators, they each correspond to a different perspective on socioeconomic inequalities in health as they weight absolute inequalities differently. For binary variables, C , W , and E all condition the level of absolute inequality on the most unequal society, but they differ in their definitions of that state. C answers the question of how far the society is from a state where the richest individual is in possession of all health units in the society (without considering the upper bound of the variable). Conversely, W and E acknowledge the boundedness of the health variable; W answers the question of how far the society is, given its overall level of health, from a state where only the individuals at the top of the income distribution are healthy, while E answers the question of how far the society is from a state where only the upper 50% of the income distribution are healthy, independent of prevalence.

Overall, the preferred index depends on the desired value judgment. If we are interested in absolute inequalities, E is the preferred index. For a relative value judgment, however, the issue is blurred; unless we relax the mirror condition (i.e. we have a priori reasons to adopt the implicit value judgments of relative inequalities measured in either health or ill-health), it is futile to discuss relative value judgments for bounded variables in the same way as for unbounded health variables. Focusing on the definition of the most unequal society, we have illustrated how Wagstaff, by normalizing C , created an index that is neither relative nor absolute, but a compromise between relative inequality in health and relative inequality in ill-health.

For relative inequality in health, the importance of absolute inequality decreases as the prevalence of health increases, while for relative inequality in ill-health, the importance of absolute inequality increases as the prevalence of health increases. Combining the two perspectives in the manner of W , adding together the level of relative inequality in health and ill-health, causes a seemingly counterintuitive behavior of the index. First, the importance of absolute inequality decreases as the level of health increases, but only up to the point of $\mu_h = 0.5$. Second, W may increase even if absolute differences in health decrease and relative differences remain unchanged.

One may, as do Erreygers and Van Ourti (2011a,b), perceive this inconsistency as undesirable. But if our initial intention was to measure relative inequality, reverting to an absolute index because we cannot decide upon a relative perspective is not necessarily superior to using a mirror relative index. Conversely, if it is possible to normatively accept a compromise between the two relative perspectives (i.e. accepting that the counterintuitive behavior of such an index is a result of this compromise), then using a mirror relative index represents an ethically defensible position that is a satisfactory alternative to using an absolute index.

Moreover, we acknowledge the compelling technical simplicity of E . Unlike W , E satisfies *linearity*, *convergence*, and *monotonicity*. All three properties make it easier to interpret and anticipate how health changes affect the index. Nevertheless, they are a result of level independence and the particular definition of the most unequal society. Thus, these properties are only desirable if one wants to capture an absolute value judgment.

Following our empirical results, we conclude that because comparisons between contexts are affected by the choice of index, the discussion of which index to use is important and not simply a matter of semantics. Therefore, we call for researchers and practitioners to consider their choice of index seriously and to reflect critically on which value judgment to impose when evaluating health inequalities. For policy makers, these findings are important because the results of different indices may call for different strategies.

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Appendix A.

A.1. The most unequal society and the weight of absolute inequalities

We can express W , E , and C as ratios between V of the observed state and V of the state in which the inequalities are maximized according to the definition of each respective index ($V^{max.I}$). Using the definitions from Section 2, we let h_i be a binary indicator of

good health and express V as:

$$\begin{aligned} V &= \frac{2}{n^2} \sum_{i=1}^n z_i h_i = \frac{2}{n^2} \sum_{i=1}^n \left(\frac{n+1}{2} - \lambda_i \right) h_i \\ &= \sum_{i=1}^n \frac{h_i}{n} + \sum_{i=1}^n \frac{h_i}{n^2} + \sum_{i=1}^n \frac{2\lambda_i h_i}{n^2} \end{aligned}$$

If, in accordance with W 's definition of the most unequal society, we let only the richest K individuals be in good health, where $K = \sum_{i=1}^n h_i$ then $\sum_{i=1}^n \lambda_i h_i = K(K+1)/2$ and $V^{max.W}$ equals:

$$V^{max.W} = \frac{K}{n} + \frac{K}{n^2} - \frac{2K(K+1)}{2n^2} = \frac{K}{n} \left(1 - \frac{K}{n} \right) = \mu_h (1 - \mu_h)$$

Thus, we can express W as:

$$W = \frac{2}{n^2(1-\mu_h)\mu_h} \sum_{i=1}^n z_i h_i = \frac{1}{(1-\mu_h)\mu_h} V = \frac{V}{V^{max.W}}$$

If, in accordance with E 's definition of the most unequal society, we let $K=0.5n$ (i.e. only the richest 50% of the individuals are in good health), then $V^{max.E}$ equals:

$$V^{max.E} = \frac{K}{n} \left(1 - \frac{K}{n} \right) = 0.25$$

Thus, we can express E as:

$$E = \frac{8}{n^2 \mu_h} \sum_{i=1}^n z_i h_i = \frac{1}{0.25} V = \frac{V}{V^{max.E}}$$

For C , the most unequal society is defined as a state where the richest individual possesses all health units in the society; that is, $h_i = n\mu_h$ for i such that $\lambda_i = 1$ and $h_i = 0$ for everyone else. Thus, $V^{max.C}$ equals:

$$\begin{aligned} V^{max.C} &= \frac{2}{n^2} \sum_{i=1}^n \left(\frac{n+1}{2} - \lambda_i \right) h_i \\ &= \frac{2}{n^2} \left(\frac{n+1}{2} - 1 \right) n\mu_h = \mu_h \left(1 - \frac{1}{n} \right). \end{aligned}$$

For a large enough n , $V^{max.C}$ tends to μ_h and we can express C as:

$$C = \frac{2}{n^2 \mu_h} \sum_{i=1}^n z_i h_i = \frac{1}{\mu_h} V = \frac{V}{V^{max.C}}$$

A.2. Mirror relativity

Definition. A rank dependent index $I(h)$ is mirror relative if

$$I(h) = B[C(h) - C(1-h)]$$

where B is a positive constant (i.e. a mirror relative index sums the level of relative inequality in health and the level of relative inequality in ill-health into the same index).

Proposition 1. A rank dependent index $I(h)$ is mirror relative if and only if

- (1) For an equiproportionate change in health from h^* to βh^* , $I(h^*)$ changes by the same magnitude as a rank-dependent index that is relative from the perspective of ill-health, i.e.

$$\frac{\partial l(\beta h^*)}{\partial \beta} = - \frac{\partial \left(\frac{A}{1-\beta\mu_h} \sum_{i=1}^n z_i(1-\beta h_i^*) \right)}{\partial \beta} \text{ for all admissible } \beta, \text{ where } A \text{ is a positive constant.}$$

(2) The normalization function f is a strictly convex function in μ_h and symmetric around $\mu_h = 0.5$.

Proof of Proposition 1. Let $I(h) = f(\mu_h, n) \sum_{i=1}^n z_i h_i$ be a mirror relative index, i.e. $I(h) = B[C(h) - C(1-h)]$, where B is a positive constant. Let $A = (2B/n^2)$, then $I(h) = (A/\mu_h) \sum_{i=1}^n z_i h_i - [A/(1-\mu_h)] \sum_{i=1}^n z_i(1-h_i) = [A/\mu_h(1-\mu_h)] \sum_{i=1}^n z_i h_i$. To evaluate equiproportionate changes of health, we consider the ratio-scaled projection of h , i.e. h^* . As $\partial \left((A/\beta\mu_h) \sum_{i=1}^n z_i(\beta h_i^*) \right) / \partial \beta = 0$ it follows that

$$\frac{\partial l(\beta h^*)}{\partial \beta} = \frac{\partial \left(\frac{A}{\beta\mu_h} \sum_{i=1}^n z_i(\beta h_i^*) - \frac{A}{(1-\beta\mu_h)} \sum_{i=1}^n z_i(1-\beta h_i^*) \right)}{\partial \beta} = - \frac{\partial \left(\frac{A}{(1-\beta\mu_h)} \sum_{i=1}^n z_i(1-\beta h_i^*) \right)}{\partial \beta}. \text{ That is (1) holds. As } f(\mu_h) = A/\mu_h(1-\mu_h) = f(1-\mu_h),$$

$\frac{\partial f(\mu_h)}{\partial \mu_h} = A(2\mu_h - 1)/[\mu_h(1-\mu_h)]^2 \begin{cases} = 0 & \text{for } \mu_h = 0 \\ > 0 & \text{for } \mu_h > 0.5, \text{ and} \\ < 0 & \text{for } \mu_h < 0.5 \end{cases}$
 $\frac{\partial^2 f(\mu_h)}{\partial \mu_h^2} = -2A(1-3\mu_h+3\mu_h^2)/[\mu_h(\mu_h-1)]^3 = (2A/\mu_h^3) - (2A/(\mu_h-1)^3) > 0$ for $0 < \mu_h < 1$, the normalization function f is a strictly convex function symmetric around $\mu_h = 0.5$. That is, (2) holds. Thus, if $I(h)$ is a mirror relative index, (1) and (2) hold.

Let (1) $\partial l(\beta h^*)/\partial \beta = -\partial \left([A/(1-\beta\mu_h)] \sum_{i=1}^n z_i(1-\beta h_i^*) \right) / \partial \beta$ for all admissible β and let (2) the normalization function f be a strictly convex function symmetric around 0.5. Since $\sum_{i=1}^n z_i(1-\beta h_i^*) = -\beta \sum_{i=1}^n z_i h_i^*$, (1) implies that $\partial \beta f(\beta\mu_h) / \partial \beta = A/(\beta\mu_h - 1)^2$. To find $f(\mu_h)$, take the anti-derivative of $\partial \beta f(\beta\mu_h) / \partial \beta$:

$$\beta f(\beta\mu_h) = \int \frac{\partial \beta f(\beta\mu_h)}{\partial \beta} d\beta = \int \frac{A}{(\beta\mu_h - 1)^2} d\beta = \frac{A}{\mu_h(1-\mu_h\beta)} + K,$$

where K is a constant. Let $K = \beta g(\beta\mu_h)$, where $g(\mu_h)$ is a function of μ_h , then

$$\beta f(\beta\mu_h) = \frac{\beta A}{\beta\mu_h(1-\beta\mu_h)} + \beta g(\beta\mu_h).$$

K is a constant (i.e. not a function of β) if and only if $g(\mu_h) = D/\mu_h$, where D is an arbitrary constant, i.e. $g(\mu_h)$ is either equal to zero or a function of μ_h that is homogenous of degree -1 . Thus $f(\mu_h) = A/\mu_h(1-\mu_h) + D/\mu_h$. Let $D \neq 0$, then $g(\mu_h) = D/\mu_h \neq D/(1-\mu_h) = g(1-\mu_h)$ and $f(\mu_h) = A/\mu_h(1-\mu_h) + g(\mu_h) \neq A/\mu_h(1-\mu_h) + g(1-\mu_h) = f(1-\mu_h)$, which is a contradiction as it violates (2). Thus, if (1) and (2) hold, $D = 0$, $f(\mu_h) = A/\mu_h(1-\mu_h)$, and $I(h) = f(\mu_h) \sum_{i=1}^n z_i h_i = [A/\mu_h + A/(1-\mu_h)] \sum_{i=1}^n z_i h_i = (A/\mu_h) \sum_{i=1}^n z_i h_i - [A/(1-\mu_h)] \sum_{i=1}^n z_i(1-h_i)$, i.e. $I(h)$ is mirror relative.

Proposition 2. A rank dependent index $I(h)$ is mirror relative if and only if:

$$\varepsilon(\mu_h) = \frac{\partial f(\mu_h, n)}{\partial \mu_h} \frac{\mu_h}{f(\mu_h, n)} = -1 + \frac{\mu_h}{1-\mu_h} = \varepsilon^{C(h)}(\mu_h) + \eta^{C(1-h)}(\mu_h)$$

that is, the elasticity of the normalization function equals the sum of the elasticities, with respect to h , of the normalization functions for $C(h)$ and $C(1-h)$.

Proof of Proposition 2. Let $I(h) = f(\mu_h, n) \sum_{i=1}^n z_i h_i$ be a mirror relative index, i.e. $I(h) = B[C(h) - C(1-h)]$, where B is a positive constant. Let $A = 2B/n^2$, then $I(h) = (A/\mu_h) \sum_{i=1}^n z_i h_i - [A/(1-\mu_h)] \sum_{i=1}^n z_i(1-h_i) = [A/\mu_h(1-\mu_h)] \sum_{i=1}^n z_i h_i$. Then,

$\varepsilon(\mu_h) = (\partial f(\mu_h, n) / \partial \mu_h) (\mu_h / f(\mu_h, n)) = (2\mu_h - 1) / (1 - \mu_h) = -1 + \mu_h / (1 - \mu_h)$. Solving the differential equation $\varepsilon(\mu_h) = [\partial f(\mu_h, n) / \partial \mu_h] (\mu_h / f(\mu_h, n)) = -1 + \mu_h / (1 - \mu_h)$ shows that $f(\mu_h) = A/\mu_h(1-\mu_h)$ is the single solution. Thus, $I(h)$ is mirror relative if and only if $\varepsilon(\mu_h) = -1 + \mu_h / (1 - \mu_h)$.

A.3. Individual health changes

Let N represent a given population. Consider a health change of m individuals represented by the set $M \subseteq N$. If these individuals enter a state of good health, then E changes as

$$\Delta E = \frac{1}{V^{max-E}} \Delta V = \frac{8}{n^2} \sum_{j \in M} z_j h_j$$

Thus, ΔE depends only on the socioeconomic rank of the additional individual's changing health state and E satisfies monotonicity. As W does not satisfy level independence the increased prevalence affects the most unequal society and the change in W equals

$$\begin{aligned} \Delta W &= \frac{1}{V^{max-W}} \Delta V + \Delta \left(\frac{1}{V^{max-W}} \right) V \\ &= \frac{1}{[1 - (\mu_h + (m/n))(\mu_h + (m/n))]} \frac{2}{n^2} \sum_{j \in M} z_j h_j \\ &\quad + \left[\frac{1}{\mu_h - \mu_h^2 + (m/n)(1 - 2\mu_h - (m/n))} - \frac{1}{\mu_h - \mu_h^2} \right] \\ &\quad \frac{2}{n^2} \sum_{i \in N-M} z_i h_i \end{aligned}$$

The first part equals the change in actual inequalities induced by the additional m individuals entering a state of good health (i.e. ΔV) weighted by the most unequal society with the new prevalence $(\mu_h + (m/n))$ (i.e. V_1^{max-W}). This part, like E , is always monotonic. The sign and size of the second part depend on both the initial absolute inequalities (i.e. V) and the change in the weight ($\Delta(1/V^{max-W})$) induced by the increased prevalence. The sign of $\Delta(1/V^{max-W})$ is negative if $\mu_h < (n-m)/2n$ and positive if $\mu_h > (n-m)/2n$. As this second part may be of the opposite sign and exceed the first, W is not monotonic.

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