



2nd International Conference ‘Economic Scientific Research - Theoretical, Empirical and Practical Approaches’, ESPERA 2014, 13-14 November 2014, Bucharest, Romania

## Impact of collinearity on estimated parameters of CES production function

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### Abstract

In many cases, the estimation of the parameters of CES production function is made with the help of Kmenta method. The respective method is frequently criticized especially because of two reasons: the approximation of the parameters is very near on the case of Cobb- Douglas production function and the impact of collinearity on the estimated parameters is usually high. This paper proposes a method to quantify the respective impact considering the values of coefficients of collinear refraction. The proposed improvements in the methodology of estimation and interpretation of CES production function parameters are applied in case of Romania during the period (1960-1979). The respective estimations offer an opportunity to reveal some features of production factors efficiency and their substitution during one of the important period for the base industrial structure building in Romania.

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Selection and/or peer-review under responsibility of the Scientific Committee of ESPERA 2014

*Keywords:* coefficient of collinear refraction, returns to scale, economic growth, production factors substitution

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### 1. Introduction. Definition of CES production function.

In the long run, the economic growth takes place in the context of the substitution between production factors. In the economic literature, the most studied type of production factors substitution is the one between the labour (L)

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and fixed capital (FK). The experiences accumulated during the industrialization period show that in the long run, the stock of fixed capital tend to increase continuously. The number of employed persons (labour force) tend to increase slower than the stock of fixed capital or even to decrease in some economic branches as a consequence of labour-saving technical change. Therefore, we may speak about two types of substitution of labour force by fixed capital, namely: 1) **relative substitution** when both production factors allocated quantities tend to increase and 2) **absolute substitution**, when the stock of fixed capital increase, while number of employed persons diminish.

The substitution between production factors is necessary to be studied from two points of view: a) the dynamics of the analyzed production factors and b) the evolution of the economic efficiency in the context of respective production factors substitution.

An indicator of the substitution process is the evolution of (fixed) capital/labour (employed persons) ratio. We may admit that as the above-mentioned ratio increase, the substitution process is intensified. But, on the other hand, the ratio (fixed) capital/ labour (employed persons) shows only partially the dynamics of production factors substitution. Another indicator of the production factors substitution is the elasticity of substitution ( $\sigma$ ), proposed by J.R Hicks in 1932. The elasticity of substitution is defined as the relative change of the ratio (fixed) capital /labour as a consequence of a modification with 1% of the marginal rate of substitution between the respective production factors, i.e.:

$$\sigma = \frac{\partial(FK / L)}{(FK / L)} : \frac{\partial\left(\frac{\partial FK}{\partial L}\right)}{\left(\frac{\partial FK}{\partial L}\right)} \quad (1)$$

Elasticity of substitution reveals the easiness of the production factors substitution. If the production factors are perfectly complementary, elasticity of substitution is equal with zero. If the production factors are perfectly substitutable, elasticity of substitution is infinite.

Due to the substitution process, the productivity of substituted production factor (labour) increase faster than the substituting production factor (fixed capital) one. Therefore, it is possible to obtain an increase of the substituted production factor productivity in the context of a sensible decrease of the substituting production factor productivity. Consequently, considering only the dynamics of a single production factor productivity may be not relevant for the appreciation of the efficiency of the economic activity.

Under these conditions, a solution is to use the notion of return to scale. Production factors substitution process may be considered as an efficient one if returns to scale are bigger than one. The mentioned-above indicator is usually associated with the Cobb-Douglas production function. The respective production function assumes that elasticity of substitution is equal with one. This means that any variation in the relative prices structure determinates a variation with the same amplitude in the intensity of the use of the two considered production factors.

Relaxation of restriction related to the size of elasticity of substitution created conditions to define (K. Arrow, H. Chenary, B. Minhas, R. Solow, 1961) the CES production function (with constant elasticity of substitution).

CES production function is expressed as:

$$\ln Y = \ln A_3 + \left(\frac{\mu \cdot \sigma}{1 - \sigma}\right) \cdot \ln(\delta \cdot L^{\frac{\sigma-1}{\sigma}} + (1 - \delta) \cdot FK^{\frac{\sigma-1}{\sigma}}) \quad (2)$$

where:

$\ln Y$  = natural logarithm of index of the output

$\ln A_3$  = natural logarithm of the residual factor

$\mu$  = return to scale

$\sigma$  = elasticity of substitution

$\delta$  = contribution of the labour (employed population) in obtaining of the output  
 L = index of the employed population  
 FK = index of (fixed) capital

## 2. Problems concerning the estimation of the parameters of CES production function

The complicated form of CES production function creates a series of problems when estimations are made. The form of CES production function presented in formula (2) cannot be linearized and estimated with the help of OLS method. Among solutions proposed for surpassing the respective difficulty, the one formulated by J. Kmenta (1967) is frequently used. Kmenta method approximate CES production function by developing in Taylor series the respective production function around the value  $\sigma = 1$ . Therefore, it is possible to obtain a production function defined by the following formula:

$$\ln Y = \ln A_3 + \alpha_3 \cdot \ln L + \beta_3 \cdot \ln FK + \gamma_3 \cdot \ln^2 (FK / L) \quad (3)$$

Parameters of respective production function may be estimated with the help of OLS method. Also, it is to be mentioned that Kmenta production function is a particular case of translog production function defined in 1973 by Christensen, Jorgensen and Lau.

The relationships between the parameters of Kmenta function production function and those of CES production function are:

$$\mu = \alpha_3 + \beta_3 \quad (4), \quad \delta = \frac{\alpha_3}{\alpha_3 + \beta_3} \quad (5) \quad \sigma = \frac{1}{1 - 2 \cdot \gamma_3 \cdot \left( \frac{1}{\alpha_3} + \frac{1}{\beta_3} \right)} \quad (6)$$

Kmenta approximation of CES production function has both costs and benefits. The main benefits are the easiness of estimation of parameters and their logical interpretation. The costs are related mainly to: a) the fact that estimated parameters are different if the computations are made in absolute values or in indices (Z. Grilichs, V Ringstad, 1971), b) the best results are theoretically obtained if the elasticity of substitution is near the case of Cobb-Douglas production function, when  $\sigma = 1$ . If departure of CES production function from Cobb-Douglas particular case is high, the estimated value of  $\sigma$  tends to loose of its relevance, c) because a quadratic term is used in estimations, the results obtained are sensibly influenced by collinearity.

Practical experiences show that harmful collinearity frequently occurs when Kmenta method is used and therefore feasibility of estimations may be questionable. Consequently, it is important to quantify the impact of collinearity on values of parameters having in view some algebraical properties of estimation method.

## 3. Quantification of impact of collinearity on the estimated parameters of CES production function when Kmenta method is used

In case of a multiple linear regression, if OLS method is used, it is possible to emphasize the impact of collinearity on the estimated values of parameters. Therefore, considering F. M. Pavelescu (1986) and F. M. Pavelescu (2005) the estimated values of  $\alpha_3$ ,  $\beta_3$  and  $\gamma_3$  may be written as:

$$\alpha_3 = \alpha_1 \cdot T_{3\alpha} \quad (7), \quad \beta_3 = \beta_1 \cdot T_{3\beta} \quad (8), \quad \gamma_3 = \gamma_1 \cdot T_{3\gamma} \quad (9),$$

where:

$\alpha_1, \beta_1$  and  $\gamma_1$  are the estimated proper values of parameters  $\alpha_3, \beta_3$  and  $\gamma_3$

$T_{3\alpha}, T_{3\beta}$  and  $T_{3\gamma}$  are the coefficients of collinear refraction

N.B. The estimated proper values of parameters are obtained when simple linear regressions between dependent and analyzed explanatory variable are run. In our case, we have:

$$\alpha_1 = \frac{\text{cov}(\ln Y, \ln L)}{D^2(\ln L)} \quad (10), \quad \beta_1 = \frac{\text{cov}(\ln Y, \ln FK)}{D^2(\ln FK)} \quad (11), \quad \gamma_1 = \frac{\text{cov}(\ln Y, \ln^2(FK/L))}{D^2(\ln^2(FK/L))} \quad (12),$$

where:

$\text{cov}(\ln Y, \ln L), \text{cov}(\ln Y, \ln FK), \text{cov}(\ln Y, \ln^2(FK/L))$  = covariance between the logarithm of output and the logarithm of considered explanatory variable

According to F. M. Pavelescu (2014b), coefficients of collinear refraction<sup>1</sup> ( $T_{3k}$ ) may be written as:

$$T_{3k} = \frac{1 - p_{jk}^{wmed} \cdot R_{2k}^2}{1 - R_{2k}^2} \quad (13) \quad \text{where:} \quad p_{jk}^{wmed} = \text{weighted arithmetical mean of ratios } p_{jk}$$

$$p_{jk} = \frac{R(x_j; y)}{R(x_j; x_k) \cdot R(x_k; y)} \quad (14), \quad \text{where:}$$

$R(x_j, y)$  = Pearson coefficient of correlation between explanatory variable  $x_j$  and dependent variable.

$R(x_j, x_k)$  = Pearson coefficient of correlation between explanatory variable  $x_j$  and explanatory variable  $x_k$ .

$R(x_k, y)$  = Pearson coefficient of correlation between explanatory variable  $x_k$  and dependent variable.

$$R_{xk}^2 = \text{Coefficient of determination of linear regression } x_k = A_{xk} + \sum_{j=1}^{n-1} x_j \quad (j \neq k)$$

Therefore, the estimated parameters of CES production function may be express as:

$$\mu = \alpha_1 \cdot T_{3\alpha} + \beta_1 \cdot T_{3\beta} \quad (15) \quad \delta = \frac{\alpha_1 \cdot T_{3\alpha}}{\alpha_1 \cdot T_{3\alpha} + \beta_1 \cdot T_{3\beta}} \quad (16)$$

$$\sigma = \frac{1}{1 - 2 \cdot (\gamma_1 \cdot T_{3\gamma}) \cdot \left( \frac{1}{\alpha_1 \cdot T_{3\alpha}} + \frac{1}{\beta_1 \cdot T_{3\beta}} \right)} \quad (17)$$

In these conditions, it is possible to detect the impact of the collinearity on the estimated values of CES production function, considering the three coefficients of collinear refraction.

<sup>1</sup> It is to be mentioned that in F M Pavelescu (2014) the indicator  $T_{3k}$  is named "coefficient of alignment to collinearity hazard". But the respective name given to the respective coefficient is too long and in a way unclear. In fact, the respective coefficient plays a role similar to index of refraction of light when the wave of light passes through different medium.

Therefore, the estimated return to scale ( $\mu$ ) can be written as:

$$\mu = (\alpha_1 + \beta_1) \cdot \text{wam} (T_{3\alpha}; T_{3\beta}), \quad (18) \text{ where:}$$

$\text{wam} (T_{3\alpha}; T_{3\beta}) =$  weighted arithmetical mean of  $T_{3\alpha}$  and  $T_{3\beta}$ .

Conditions to obtain feasible results in estimation of the returns to scale for a production function are that  $\alpha_1, \beta_1, T_{3\alpha}$ , and  $T_{3\beta}$  are positive. In other words, the feasible results are obtained in conditions when economic growth has taken place in the same time with an increase of the allocated quantity of both production factors<sup>†</sup> and the occurrence of the harmful collinearity is avoided<sup>‡</sup>.

If the harmful collinearity does not occur, the values of  $T_{3\alpha}$  and  $T_{3\beta}$  are comprised between 0 and 1. If harmful collinearity is manifest, we have the right to reject the estimation results. Consequently, the presence of collinearity determines a decrease of estimated value for returns to scale estimated for CES production function in comparison with the sum of proper elasticities of output related the considered production factors. The impact of collinearity on estimated returns to scale is given by the value of  $\text{wam} (T_{3\alpha}; T_{3\beta})$ .

But if we have in mind that the returns to scale is estimated also in case of Cobb-Douglas production function it is possible to emphasize not only the whole impact of collinearity on estimated parameters values. We are able to detect the influence of the collinearity between logarithm of fixed capital indices and logarithm of employed population indices, on the one hand, and the influence of adding in the production function of the explanatory variable  $\ln^2 (FK/L)$ .

If we have in view the Cobb-Douglas production function we may express the estimated return to scale related to respective production function ( $\mu_{CD}$ ) as:

$$\mu_{CD} = (\alpha_1 + \beta_1) \cdot \text{wam} (T_{2\alpha}; T_{2\beta}), \quad (19) \text{ where:}$$

$\text{wam}(T_{2\alpha}; T_{2\beta})=$ weighted arithmetical mean of  $T_{2\alpha}$  and  $T_{2\beta}$  (coefficients of collinear refraction obtained in case of linear regression  $\ln Y = \ln A_2 + \alpha_2 \cdot \ln L + \beta_2 \cdot \ln FK$ ).

Therefore, we able to write:

$$\mu = (\alpha_1 + \beta_1) \cdot \text{wam} (T_{2\alpha}; T_{2\beta}) \cdot \left( \frac{\text{wam} (T_{3\alpha}; T_{3\beta})}{\text{wam} (T_{2\alpha}; T_{2\beta})} \right) \quad (20)$$

This way, it is possible to quantify the influence of collinearity in estimating the returns to scale in case of the Cobb-Douglas production function ( $\text{wam}(T_{2\alpha}; T_{2\beta})$ ).The influence of collinearity determined by adding of the explanatory variable  $\ln^2 (FK/L)$  is emphasized by the ratio ( $\text{wam} (T_{3\alpha}; T_{3\beta}) / \text{wam}(T_{2\alpha}; T_{2\beta})$ ).

The influence of collinearity on the estimated value of  $\delta$  can be similarly obtained, because it is possible to write:

$$\delta = \frac{\alpha_1}{\alpha_1 + \beta_1} \cdot \frac{T_{2\alpha}}{\text{wam} (T_{2\alpha}; T_{2\beta})} \cdot \left( \frac{T_{3\alpha}}{T_{2\alpha}} \cdot \frac{\text{wam} (T_{3\alpha}; T_{3\beta})}{\text{wam} (T_{2\alpha}; T_{2\beta})} \right) \quad (21)$$

The estimated value of elasticity of substitution can be expressed as:

<sup>†</sup> The problem of significance of estimated returns to scale and of the feasibility of respective estimations in the context of Cobb-Douglas production function is largely analyzed in F. M. Pavelescu (2014a).

<sup>‡</sup> If we consider the coefficients of collinear refraction we are able to admit that negative values of respective indicators are signals for the presence of harmful collinearity within the results obtained in estimation.

$$\sigma = \frac{1}{1 - 4 \cdot \frac{\gamma_3}{hm(\alpha_3; \beta_3)}} \quad (22), \text{ where: } hm(\alpha_3; \beta_3) = \text{harmonical mean of } \alpha_3 \text{ and } \beta_3.$$

If we consider the formulae (7), (8) and (9), formula (22) is equivalent with:

$$\sigma = \sigma_{pr} \cdot IColin\sigma \quad (23), \text{ where:}$$

$$\sigma_{pr} = \frac{1}{1 - 4 \cdot \frac{\gamma_1}{hm(\alpha_1; \beta_1)}} \quad (24)$$

$$IColin\sigma = \frac{1 - 4 \cdot \frac{\gamma_1}{hm(\alpha_1; \beta_1)}}{1 - 4 \cdot \frac{\gamma_1}{hm(\alpha_1; \beta_1)} \cdot \frac{T_{3\gamma}}{whm(T_{3\alpha}; T_{3\beta})}} \quad (25), \text{ where:}$$

$hm(\alpha_1; \beta_1)$  = harmonical mean of  $\alpha_1; \beta_1$ .

$whm(T_{3\alpha}; T_{3\beta})$  = weighted harmonical mean of  $T_{3\alpha}; T_{3\beta}$ .

Formula (24) shows interesting features of estimated values of elasticity of substitution if the phenomenon of collinearity would not occur.

If we admit the assumption that, in the long run, there is a trend of growth of both output and stock of fixed capital, and that the number of employed persons may have either a trend to increase or to decrease and that explanatory variables of Kmenta production function are strictly orthogonal, we are faced with two situations, namely:

A)  $\alpha_1 > 0, \beta_1 > 0, \gamma_1 > 0$  and

B)  $\alpha_1 < 0, \beta_1 > 0, \gamma_1 > 0$ .

Therefore, in situation A) we may find two cases for the factor  $\sigma_{pr}$  respectively:

A I)  $\sigma_{pr} > 1$ , if  $\gamma_1 < 4 \cdot hm(\alpha_1; \beta_1)$

A II)  $\sigma_{pr} < 0$  if  $\gamma_1 > 4 \cdot hm(\alpha_1; \beta_1)$

In situation B) we may find three cases for the factor  $\sigma_{pr}$ , namely:

B I)  $\sigma_{pr} > 1$ , if  $\gamma_1 < 4 \cdot hm(\alpha_1; \beta_1)$  and  $hm(\alpha_1; \beta_1) > 0$

B II)  $\sigma_{pr} < 0$  if  $\gamma_1 > 4 \cdot hm(\alpha_1; \beta_1)$  and  $hm(\alpha_1; \beta_1) > 0$

B III)  $0 < \sigma_{pr} < 1$  if  $hm(\alpha_1; \beta_1) < 0$ .

If we are faced with situations, which are in contradiction with those presented-above, this is a signal for the occurrence of harmful collinearity especially related to parameter  $\gamma$ .

The value of factor  $IColin\sigma$  is essentially determined by the ratio  $(T_{3\gamma} / whm(T_{3\alpha}; T_{3\beta}))$ . Therefore, it is possible to detect three kinds of impact on the estimated value of elasticity of substitution, namely:

1) if  $IColin\sigma > 1$ , we have  $\sigma > \sigma_{pr}$  as a general tendency. An exception may occur if multiplier effect of  $IColin\sigma$  may determinate a negative value of  $\sigma$ , in the context of a positive  $\sigma_{pr}$ , when the vertical asymptotical point is surpassed.

2) if  $0 < IColin\sigma < 1$ , we have  $\sigma < \sigma_{pr}$  as a general tendency. An exception may occur if de-multiplier effect of  $IColin\sigma$  may determinate a positive value of  $\sigma$ , in the context of a negative  $\sigma_{pr}$ , when the vertical asymptotical point is surpassed.

3) if  $\text{Icolin}\sigma < 0$ , we are in a situation of harmful collinearity. In this context, if the ratio  $(\gamma_1/\text{hm}(\alpha_1;\beta_1))$  is positive the estimated elasticity of substitution is comprised between 0 and 1. On the other hand, if the ratio  $(\gamma_1/\text{hm}(\alpha_1;\beta_1))$  is negative, the estimated elasticity of substitution may be either positive and supraunitary or negative.

#### 4. Estimation of parameters of CES production function for Romania during the period 1960-1979

Considering the data used in (F. M. Pavelescu, 2014a) for estimation of returns to scale in case of Cobb-Douglas production function<sup>§</sup> we have determined the parameters of CES production function and applied the mentioned above methodology for the interpretation of the estimation results.

The estimated Kmenta production function parameters and related Student Statistics are presented in Table 1. It is to note that Coefficient of determination ( $R^2$ ) is 0.9970.

In these conditions, we obtain:  $\mu=3.5789$ ,  $\delta=0.7193$  and  $\sigma=0.8845$ . In other words, we may detect in the analyzed period of the Romania's economy evolution the following characteristic features: a) increasing returns to scale, b) a major contribution (near 72%) of labour to the obtaining of the returns to scale, c) a positive underunitary elasticity of substitution, due to negative value of  $\gamma_3$ .

Table 1. Estimated parameters of Kmenta production function for Romania during 1960-1979 period

Parameter	Estimated value	Student Test Statistics
$\text{Ln}A_3$	0.0543	2.52672
$\alpha_3$	2.5743	0.9239
$\beta_3$	1.0047	6.4330
$\gamma_3$	-0.0472	-0.1214

The estimated values suggest an important impact of collinearity on the estimated parameters, even if we look at the Student Test statistics in case of  $\alpha_3$  and  $\gamma_3$ .

Estimation of simple regressions between output (national income) and each of the explanatory variable considered in Kmenta production function permit to obtain the proper values and coefficients of collinear refraction, i.e.:  $\alpha_1= 20.0492$  and  $T_{3\alpha}= 0.1284$ ;  $\beta_1= 1.0721$  and  $T_{3\beta}= 0.9371$ ;  $\gamma_1= 0.6840$  and  $T_{3\gamma}= -0.0690^{**}$ . This means that harmful collinearity occurs in case of explanatory variable  $\text{ln}^2(\text{FK}/\text{L})$ . Also, we may note that during the analyzed period both the productivity of labour and of fixed capital had a trend to increase. The ratio between the proper elasticities of output related to each production factor suggests a sensible increase of the ratio fixed capital/labour.

Estimation of the Cobb-Douglas production function with non-constant returns to scale led to results presented in Table 2 in the context of  $R^2=0.9891$ .

Table 2. Estimated parameters of Cobb-Douglas production for Romania during 1960-1979 period

Parameter	Estimated value	Student Test Statistics
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<sup>§</sup> It is to be mentioned that in F.M. Pavelescu (2014b) the output is the national income computed in conditions of System of Material Production, while the considered production factors are: fixed assets and employed population at the level of the whole economy.

\*\* Running of simple linear regression between the dependent variable and each of the considered explanatory variable permits to easily compute the Pearson coefficient of correlation between the dependent variable and respective explanatory variables. In our case, the mentioned-above Pearson coefficients of correlation are:  $R(\text{ln}Y;\text{ln}L)=0.9945$ ,  $R(\text{ln}Y;\text{ln}FK)= 0.9983$  and  $R(\text{ln}Y: \text{ln}^2(\text{FK}/\text{L})= 0.9623$ . The values of Pearson coefficients of correlation reveals that each of the explanatory variable is very highly correlated with the output. Also, it is important to note that the hierarchy of the size of coefficients of collinear refraction is the same with the hierarchy given by the absolute value of the Pearson coefficients of correlation between the logarithm of output and the logarithm of explanatory variable.

$\ln A_2$	0.0629	3.8419
$\alpha_2$	2.4669	0.8644
$\beta_2$	0.9622	6.3613

It is to note that coefficients of collinear refraction determined in case of Cobb-Douglas production function are:  $T_{2\alpha} = 0.1230$  and  $T_{2\beta} = 0.8974$ , meaning that output's dynamics is mainly correlated with stock of fixed capital dynamics. Consequently  $\mu_{CD} = 3.4291$ . Because  $\alpha_1 + \beta_1 = 21.1213$ , we obtained  $wam(T_{2\alpha}; T_{2\beta}) = 0.1624$ .

Adding in the production function of the explanatory variable  $\ln^2(FK/L)$  determined an increase of coefficients of collinear refraction for both above mentioned production factors. The respective increase of coefficients of collinearity is made at the cost of occurrence of negative coefficient of collinear refraction related to  $\ln^2(FK/L)$ . Consequently,  $wam(T_{3\alpha}; T_{3\beta}) = 0.1694$ . The ratio  $(wam(T_{3\alpha}; T_{3\beta}) / wam(T_{2\alpha}; T_{2\beta}))$  is equal with 1.0434 and reveals the positive influence for estimated value of returns to scale of considering the explanatory variable  $\ln^2(FK/L)$  within the production function.

Contribution of labour to returns to scale was also influenced by collinearity phenomenon. Therefore, in terms of estimated proper values of elasticities of output related to considered production factors, the contribution of labour to returns to scale (ratio  $(\alpha_1 / (\alpha_1 + \beta_1))$ ) is 94.92%. The respective contribution is 71.94% in case of Cobb-Douglas production function, being practically equal with the value obtained in case of CES production function. The above-mentioned result reveals that adding of the term  $\ln^2(FK/L)$  in linear regression equation determined an increase with practically same rate of the coefficients of collinear refraction of the production factors considered in Cobb-Douglas function.

If we have in view only the estimated proper values  $\alpha_1$ ,  $\beta_1$  and  $\gamma_1$ , we obtain:  $\sigma_{pr} = -2.9046$ . The ratio  $(T_{3\gamma} / whm(T_{3\alpha}; T_{3\beta}))$  is equal with -0.0665. Consequently,  $\text{Inflcolin}\sigma = -0,3045$ . This way it is possible to obtain the positive but underunitary value of elasticity of substitution, 0.8845, respectively.

The results obtained in case of Romania are in line with estimations related to elasticity of substitution made for other countries and the other parameters of CES production function. In many cases ((E. Miller, 2008) and (R.Klump, P.McAdam, A. Willman, 2011)) there are estimated increasing returns to scale and positive but underunitary elasticity of substitution.

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