



# Geometrical origin of entropy during inflation from the STM theory of gravity

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## ABSTRACT

Using a recently introduced 5D Riemann flat metric, we investigate the possibility of introducing dissipation in the dynamics of the inflaton field on an effective 4D FRW metric, in the framework of the STM theory of gravity.

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## 1. Introduction

A number of higher-dimensional theories such as 11D supergravity [1] and 10D superstrings [2] appeared inspired by the 5D Kaluza–Klein (KK) theory [3] in the sixties and seventies, all of them aiming at a general scheme of unification [4]. More recently, the so-called braneworld scenario has emerged, according to which our 4D spacetime is viewed as a hypersurface (the brane) isometrically embedded in an 5D space (the bulk) [5]. The original version of the KK theory assures, as a postulate, that the fifth dimension is compact. A few years ago, a non-compactified approach to KK gravity, known as Space–Time–Matter (STM) theory was proposed by Wesson and collaborators [6]. In this theory all classical physical quantities, such as matter density and pressure, are susceptible of a geometrical interpretation. Wesson's proposal also assumes that the fundamental 5D space in which our usual spacetime is embedded, should be a solution of the classical 5D vacuum Einstein equations:  $R_{AB} = 0$ . The mathematical basis of it is the Campbell's theorem [7], which ensures an embedding of 4D general relativity with sources in a 5D theory whose field equations are apparently empty. That is, the Einstein equations  $G_{\alpha\beta} = -8\pi G T_{\alpha\beta}$  (we use  $c = \hbar = 1$  units), are embedded perfectly in the Ricci-flat equations  $R_{AB} = 0$ . In simple terms, Wesson uses the fifth dimension to model matter. An alternative version of 5D gravity, which is mathematically similar, is the membrane theory. In this, gravity propagates freely in 5D, into the “bulk”, but the interactions of particles are confined to a 4D hypersurface called “brane”. Both versions of 5D general relativity are in agreement with observations. Inflationary cosmology is one of the most reliable concepts in modern cosmology. In particular, the warm inflation scenario

[8] takes into account separately, the matter and radiation energy fluctuations. In this scenario, the interaction between the inflaton field and the particles of a thermal bath with temperature  $T$  produced during inflation, provides slow-rolling of the inflaton towards the minimum of the potential  $V(\varphi)$ . Hence, in this model slow-roll conditions are physically well justified. The decay width ( $\Gamma$ ) of the produced particles grows with time, so when the inflaton approaches the minimum of the potential there is no oscillation around the minimum energetic configuration. This is due to dissipation being too large with respect to the Hubble parameter ( $\Gamma \gg H$ ).

The aim of this Letter consists to explain how dissipative dynamics of the inflaton field on a 4D Friedmann–Robertson–Walker (FRW) metric, can be geometrically induced from a dynamically foliated 5D Riemann flat metric, on which we shall define our physical vacuum. Moreover, we are interested to study how this dissipation is related to the fifth coordinate and the growth of entropy, when the equivalence principle is broken.

## 2. 5D vacuum and field dynamics

We consider the recently introduced 5D line element [9]

$$dS^2 = \psi^2 \frac{\Lambda(t)}{3} dt^2 - \psi^2 e^{2 \int \sqrt{\Lambda(t)/3} dt} dr^2 - d\psi^2, \quad (1)$$

where  $\Lambda(t)$  is a time-dependent function,  $dr^2 = dx^2 + dy^2 + dz^2$  is the 3D Euclidean metric,  $t$  is the cosmic time and  $\psi$  is the space-like extra dimension. Choosing a natural unit system the function  $\Lambda(t)$  has units of  $[\text{length}]^{-2}$ . The metric (1) is Riemann flat and hence is suitable to describe a 5D vacuum state in the context of Space–Time–Matter (STM) theory of gravity (the reader can see [10] and references therein).

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To describe the system in an apparent vacuum, we shall consider the action

$${}^{(5)}I = \int d^4x d\psi \sqrt{\left| \frac{{}^{(5)}g}{{}^{(5)}g_0} \right|} \left( \frac{{}^{(5)}R}{16\pi G} + \frac{1}{2} g^{AB} \varphi_{,A} \varphi_{,B} \right), \quad (2)$$

where  ${}^{(5)}g$  is the determinant of the covariant metric tensor  $g_{AB}$  ( $A$  and  $B$  run from 0 to 4):

$${}^{(5)}g = \psi^8 \frac{\Lambda}{3} e^{6 \int \sqrt{\Lambda/3} dt}. \quad (3)$$

For simplicity we shall consider  $\varphi$  as a classical scalar field, which is minimally coupled to gravity. The Lagrangian is purely kinetic because we describe an apparent vacuum in absence of interactions. In this sense,  $\varphi$  can be considered as a massless test field on the 5D vacuum. However, a more consistent approach would take into account the quantum nature of  $\varphi$ . The equation of motion for the field  $\varphi$  is

$$\ddot{\varphi} + \left[ 3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda} \right] \dot{\varphi} - \frac{\Lambda}{3} e^{-2 \int \sqrt{\Lambda/3} dt} \nabla^2 \varphi - \frac{\Lambda}{3} \left[ 4\psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] = 0, \quad (4)$$

which describes the dynamics of  $\varphi$  on the 5D vacuum (1).

### 3. Dissipative dynamics of $\varphi$ on a FRW metric

We shall assume that the 5D spacetime (1) is dynamically foliated on the fifth coordinate:  $\psi = \psi(t)$ , such that the effective 4D hypersurface is

$$\begin{aligned} dS^2|_{\psi=\psi(t)} &= d\sigma^2|_{\psi=\psi(t)} - \dot{\psi}^2(t) dt^2 \\ &\equiv \left[ \psi^2(t) \frac{\Lambda(t)}{3} - \dot{\psi}^2(t) \right] dt^2 \\ &\quad - \psi^2(t) e^{2 \int \sqrt{\Lambda(t)/3} dt} dr^2, \end{aligned} \quad (5)$$

with the condition  $\psi^2(t) \frac{\Lambda(t)}{3} - \dot{\psi}^2(t) > 0$ , such that  $g_{AB} U^A U^B = 1$  [ $U^A = \frac{dx^A}{ds}$  are the components of the penta-velocity]. However, we are interested to study the evolution of  $\varphi$  on the hypersurface  $d\sigma^2|_{\psi=\psi(t)}$ , which can be considered as a brane on (5):

$$d\sigma^2|_{\psi=\psi(t)} = g_{\mu\nu}(t, \vec{r}, \psi(t)) dx^\mu dx^\nu \neq dS^2|_{\psi=\psi(t)}. \quad (6)$$

Notice that the equivalence principle is broken on (6):  $g_{AB} U^A U^B \neq 1$ . We can define  $\rho = \rho_0 + \Delta\rho$  and  $p = p_0 + \Delta p$ , the energy density and the pressure on  $d\sigma^2|_{\psi=\psi(t)}$  [described by (6)] in (5): such that  $\rho_0(t)$  and  $p_0(t)$  are the energy density and the pressure on the effective 4D hypersurface  $dS^2|_{\psi=\psi(t)}$  in (5). Furthermore, the system describes an adiabatic expansion on (5), so that (from the thermodynamical point of view) it can be considered as a closed system

$$\frac{d}{dt} [\rho_0 a^3(t)] + p_0 \frac{d}{dt} [a^3(t)] = 0. \quad (7)$$

Hence,  $\Delta\rho$  and  $\Delta p$  comply with the following equation on (6):

$$\Delta\dot{\rho} + 3H(\Delta\rho + \Delta p) = \frac{T}{a^3(t)} \dot{S}. \quad (8)$$

Here,  $a(t) = \psi(t) e^{\int \sqrt{\Lambda/3} dt}$ ,  $H(t) = \dot{a}/a$ ,  $T$  is the temperature of the system on  $d\sigma^2|_{\psi=\psi(t)}$  and  $S$  is its entropy. We can define  $\gamma = 1 + \frac{\Delta p}{\Delta\rho}$ , so that Eq. (8) can be rewritten as

$$\Delta\dot{\rho} + 3H\gamma\Delta\rho = \frac{T}{a^3(t)} \dot{S}. \quad (9)$$

A very interesting case, of particular interest is  $\frac{\Delta p}{\Delta\rho} = 1/3$ . In this particular case we can identify  $\Delta\rho$  and  $\Delta p$  with the radiation energy density and its pressure:  $\Delta\rho \equiv \rho_r$  and  $\Delta p \equiv p_r$ . In this case  $\gamma = 4/3$  and the system radiates

$$\dot{\rho}_r + 4H\rho_r = \delta, \quad (10)$$

where the interaction  $\delta \equiv \frac{T}{a^3(t)} \dot{S} > 0$  is related with the variation of entropy  $S$ . Notice that the entropy  $S$  increases with time, so that both sides in (10) are positives.

The Lagrange equation describes a non-conservative system:

$$\frac{\partial^{(4)}\mathcal{L}}{\partial\varphi} - \frac{\partial}{\partial x^\mu} \frac{\partial^{(4)}\mathcal{L}}{\partial\varphi_{,\mu}} = F_{\text{ext}}, \quad (11)$$

where  $F_{\text{ext}} \sim \dot{\varphi}$  describes a non-conservative term, which can be related with a Lagrangian interaction. The origin of this term can be interpreted on the metric (6) as due to the interaction of  $\varphi$  with other fields in a thermal bath with temperature  $T$ . This term could be the origin of energy dissipated by the  $\varphi$  field into a thermalized bath.

### 4. An example

To illustrate the earlier results, we can consider the *particular foliation* where  $\psi^2(t) = 3/\Lambda(t)$ . In this case the effective 4D metric (5) is

$$\begin{aligned} dS^2|_{\psi(t)=\sqrt{3/\Lambda(t)}} &= \left[ 1 - \frac{3}{\Lambda(t)} \left( \frac{\dot{\Lambda}(t)}{2\Lambda(t)} \right)^2 \right] dt^2 \\ &\quad - \frac{3}{\Lambda(t)} e^{2 \int \sqrt{\Lambda(t)/3} dt} dr^2, \end{aligned} \quad (12)$$

so that

$$d\sigma^2|_{\psi(t)=\sqrt{3/\Lambda(t)}} = d\sigma^2|_{\psi(t)=\sqrt{3/\Lambda(t)}} - \frac{3}{\Lambda(t)} \left( \frac{\dot{\Lambda}(t)}{2\Lambda(t)} \right)^2 dt^2,$$

with

$$\begin{aligned} d\sigma^2|_{\psi(t)=\sqrt{3/\Lambda(t)}} &= dt^2 - \frac{3}{\Lambda(t)} e^{2 \int \sqrt{\Lambda(t)/3} dt} dr^2 \\ &\equiv dt^2 - a^2(t) dr^2. \end{aligned} \quad (13)$$

From the thermodynamical point of view, the hypersurface described by the brane  $d\sigma^2|_{\psi(t)=\sqrt{3/\Lambda(t)}}$  can be considered as an open system with respect to the closed one  $dS^2|_{\psi(t)=\sqrt{3/\Lambda(t)}}$  in (12). We shall consider that (13) is our physical spacetime.

The equation of motion for the field  $\varphi(t, \vec{r})$  on the metric (13) [induced from the 5D vacuum defined by the action (8), on the metric (1)], is

$$\begin{aligned} \ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} - \frac{1}{a^2}\nabla_r^2\varphi \\ - \frac{\Lambda(t)}{3} \left[ 4\psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] \Big|_{\psi(t)=\sqrt{3/\Lambda(t)}} = -\frac{\delta}{\varphi}, \end{aligned} \quad (14)$$

where the interaction on the right-hand side of (14) has its origin in the temporal dependence of the fifth coordinate on the brane (13). In other words, non-conservative terms in the dynamics of the equation of motion for  $\varphi$  are induced by time dependent terms of  $g_{tt}$  in (12) omitted in (13). For this reason, the interaction  $\delta$  is also dependent on  $\dot{\psi}$ , so that

$$\delta = 2 \frac{\dot{\psi}}{\psi} \dot{\varphi}^2 = \frac{T}{a^3(t)} \dot{S} > 0. \quad (15)$$

Therefore, for a given temperature  $T$ , and a given foliation  $\psi(t) = \sqrt{3/\Lambda(t)}$ , we can obtain the temporal evolution of entropy as a function of the fifth coordinate:

$$\dot{S} = - \left( \frac{\dot{\Lambda}(t)}{\Lambda(t)} \right) \frac{a^3}{T} \dot{\varphi}^2 > 0, \quad (16)$$

which means that for  $\dot{\psi}(t) = -\sqrt{3/\Lambda(t)}(\frac{\dot{\Lambda}(t)}{2\Lambda(t)}) > 0$ , the function  $\Lambda(t)$  decreases with time:  $\dot{\Lambda}(t) < 0$ . Finally, Eq. (14), which describes the dynamics of  $\varphi$  on the FRW metric (13) from the foliated  $\psi(t) = (3/\Lambda(t))^{1/2}$  Riemann flat metric (1), is

$$\ddot{\varphi} + \left[ 3\frac{\dot{a}}{a} - \frac{\dot{\Lambda}(t)}{\Lambda(t)} \right] \dot{\varphi} - \frac{1}{a^2} \nabla_r^2 \varphi - \frac{\Lambda(t)}{3} \left[ 4\psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] \Big|_{\psi(t)=\sqrt{3/\Lambda(t)}} = 0, \quad (17)$$

where the term  $\Gamma(t)\dot{\varphi} = -\frac{\dot{\Lambda}(t)}{\Lambda(t)}\dot{\varphi}$  has a purely dissipative interpretation on the brane (13) and describes the energy dissipated by the inflaton field into a thermalized radiation bath. Notice that the factor  $[3\frac{\dot{a}}{a} - \frac{\dot{\Lambda}(t)}{\Lambda(t)}]$  in (17) is the same that whole of Eq. (4). Notice that Eqs. (10) and (17) are the same equations that describe warm [8] and fresh [11] inflationary scenarios with a Yukawa self-interaction of  $\varphi$ :  $\delta = \Gamma\dot{\varphi}^2$  and a width  $\varphi$ -decay:  $\Gamma = 2\frac{\dot{\psi}(t)}{\psi(t)}|_{\psi(t)=\sqrt{3/\Lambda(t)}} = -\frac{\dot{\Lambda}(t)}{\Lambda(t)}$ . In these scenarios, accelerated expansion and radiation of energy are produced together during the inflationary stage and the Einstein equations are

$$H(t)^2 = \frac{8\pi G}{3} \rho(t), \quad (18)$$

$$-(3H(t)^2 + 2\dot{H}(t)) = 8\pi G p(t), \quad (19)$$

where  $\rho(t)$  and  $p(t)$  are respectively given by

$$\rho(t) = \frac{\dot{\varphi}^2(t)}{2} + V(\varphi) + \rho_r(t), \quad (20)$$

$$p(t) = \frac{\dot{\varphi}^2(t)}{2} - V(\varphi) + \frac{\rho_r(t)}{3}, \quad (21)$$

with  $V(\varphi) = -\frac{1}{2}(\frac{\partial \varphi}{\partial \psi})^2|_{\psi(t)=\sqrt{3/\Lambda(t)}}$  and  $H(t) = \dot{a}(t)/a(t)$  is the Hubble parameter of the universe on the FRW metric (13).

Using the fact that the red-shifted temperature evolves as  $T(t) = T_0 [a_0/a(t)]$ , Eq. (16) can be rewritten as

$$\dot{S} = \frac{\Gamma(t)a^4(t)}{T_0 a_0} \dot{\varphi}^2 > 0, \quad (22)$$

where  $T_0$  is the background temperature at  $t_0 < t$  and  $a_0$  is the scale factor at this moment. This result is a generalization for 3D spatially flat FRW metrics of whole obtained using finite temperature quantum field theory by Hosoya and Sakagami [12].

Finally, in order to illustrate our calculations, we can work an example in which  $\Lambda(t) = 3n^2/t^2$ . In this case  $\Gamma = -\dot{\Lambda}(t)/\Lambda(t) = 2/t$  and the Hubble parameter is  $H(t) = (n+1)/t$  for an accelerating universe with a scale factor  $a(t) \sim t^{n+1}$ . This implies that dissipation should be dominant for  $\Gamma(t)/\sqrt{\Lambda(t)/3} > 1$  [12], that is for  $n < 1$ . In other words, dissipation should be very important in models of inflation where the power of expansion of the universe  $(n+1)$  is moderated:  $1 < (n+1) < 2$ .

## 5. Final remarks

We have shown how dissipative dynamics can be induced (from a 5D vacuum state) in the inflaton field, which evolves on an effective 4D FRW metric during inflation. A very interesting fact that results from the analysis is that the entropy increases with time when  $\dot{\psi}(t) > 0$ . Furthermore, the expression (22) provide us  $\dot{S}(t)$  as a function of geometrical quantities and the initial temperature. From the point of view of General Relativity, the emergence of dissipation in the dynamics of  $\varphi$  is a consequence of that the equivalence principle is broken on the effective 4D FRW metric:  $g_{AB}U^A U^B \neq 1$ . This is also the origin of entropy on the FRW metric (12) [see Eq. (22) for  $\dot{S}(t)$ , which becomes zero for  $\Gamma = 0$ ], which can be considered as a 4D brane of the 5D Riemann-flat bulk (1). In this sense, dissipative dynamics of the inflaton field during inflation in our 4D brane (i.e., our spacetime) can be considered as indirect evidence of extra dimensions's existence. However, the system evolves adiabatically on the metric (12), which is comes from a  $\psi(t) = \sqrt{3/\Lambda(t)}$ -foliation on the 5D metric (or bulk) (1).

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